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GEOMETRICAL DRAWING AND DESIGN.

## GEOMETRICAL

## DRAWING AND DESIGN

BY

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## PREFACE.

A COURSE of geometrical drawing or practical geometry provides a valuable preliminary training for so many handicrafts and professions that it must be regarded as essential to all students whose work is to be adapted to modern requirements. The Engineer, the Architect, the Soldier, and the Statistician, all have recourse to the assistance of practical geometry to solve their problems or to explain their methods. Every day the graphic treatment of subjects is finding its application in new directions ; and to be able to delineate the proportions of any scheme places in the hand an invaluable tool for the execution of work of practical value.

The Author's complete geometrical course has now for some time been widely used for the advanced parts of the suibjent, such as Projection, Section and Interpenetration of Solids. Hence the publication of its simpler parts as an introduction to Design would seem likely to meet with favour.

The geometrical basis of ornamentation is the rational one, though youth and fancy might condemn it as a chaining of Pegasus and the curbing of imagination. It is no doubt the height of art to conceal the scheme of treatment and delight the eye with novel suggestions of development. But underlying all is the demand of Nature for order and rhythm, such as can be obtained by a study of geometrical figures and designs.

The course of work prescribed in Geometrical Drawing (Art) by the Board of Education aims at giving students the ability to construct ordinary geometrical figures, and the power to
apply these figures as the basis of ornamental and decorative work. In the preparation of the present volume these intentions have been borne in mind, but the scope of the work has not been limited by the syllabus of the subject. The greater part of the book contains an elementary course of constructive geometry suitable for all students, and the sections which show the applications of geometrical constructions to design, while of especial importance to students of applied art, are not without interest to students of science. Moreover, familiarity with the constructions in the early part of the book provides the best introduction a pupil could have to the study of formal geometry.

The Editor's thanks are due to Prof. Thos. C. Simmonds, the Headmaster of the Municipal School of Art, Derby, for his valuable advice, and to Mr. E. E. Clark, his assistant, by whom the drawings for the sections on Design have been furnished. Acknowledgment must also be made of assistance rendered by Prof. R. A Gregory in arranging the book and seeing it through the press.

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## PART I.

## PRACTICAL PLANE GEOMETRY.

## INTRODUCTION.

## Drawing Instruments and Materials.

The Drawing-board.-A very convenient size to use for ordinary purposes is half Imperial ( $23^{7} \times 16^{\prime \prime}$ ) ; it should be made of well-seasoned yellow pine, with the edges true and at right angles to each other.

The Tee-square. - This is a ruler with a cross piece or stock at the end: it is like the letter $\mathbf{T}$ in shape, hence its name. The blade should be screwed on to the stock, and not mortised into it, so as to allow of the set-squares being used up to the extreme margin of the paper, as illustrated in Fig. 1. By keeping the stock of the tee-square pressed closely against the edge of the drawing-board, we are enabled to draw lines parallel to each other.

Set-squares. - These are right-angled triangles made with given fixed angles out of thin pieces of wood or ebonite : the latter is preferable, as it is not liable to warp. The most useful angles are those of $45^{\circ}$ and $60^{\circ}$.

French Curves. - These are thin pieces of wood cut into a variety of curves. They are used for drawing fair curves, that are not arcs of circles, through a succession of points: the cycloidal curves, for instance.

Scale.-A plain scale about 6 inches long, divided into inches, with sub-divisions of eighths on one edge and tenths on the other.

Pencils.-Two degrees of hardness should be used: HH for drawing in the construction, and F for drawing in the result with a firmer line.

Drawing-paper. - This should have a hard smooth surface. Whatman's "hot-pressed" is the best for fine work ; but if the drawing has to be coloured, a damp sponge should first be drawn across the surface, to remove the gloss. Cartridge-paper of good quality is suitable for ordinary work.

The most convenient size is "Imperial" $\left(30^{\prime \prime} \times 22^{\prime}\right.$ ), which can be cut to half, or quarter Imperial, as desired.

Drawing-pins.-These should have short fine points, so as not to make large holes in the drawing-board.

Dividers.- These are used for setting off distances or dividing lines. There is a special kind made, called "hairdividers," one leg of which can be adjusted by means of a spring and screw : these are very useful for dividing lines, etc.

Ruling-pen. - This is used for inking in lines, the thickness of which is regulated by a screw. Some are made in which the nib that works against the ruler is of an extra thickness of metal: this is to prevent the nibs from closing when the pen is pressed against the ruler.

Bow-pencil.-This is a small pair of compasses with one leg constructed to hold a pencil, used for drawing circles and arcs.

Bow-pen.--This is a similar instrument to a bow-pencil, but with a ruling pen for one of its legs instead of a pencil ; it is used for inking in circles and arcs.

Note.--Both the bow-pencil and bow-pen should have hinged legs; because, when a number of circles are drawn from the same centre, they are likely to make a large hole in the paper, unless the leg used for the centre is kept perpendicular to the paper. It is also necessary to have the pen-leg as upright as possible, otherwise it has a tendency to draw uneven lines.

Compasses. -Full-sized compasses are supplied, with interchangeable arms, divider, pen and pencil, opening to 12 inches.

Indian ink should be used for inking in a drawing. It has several advantages over common ink: it dries quickly; it does not corrode the ruling-pen ; and the lines can be coloured over without their running, if a waterproof quality is used.

The most convenient is the liquid Indian ink, sold in bottles,
as it is always ready for use. The ruling-pen should be filled with Indian ink by means of an ordinary steel nib. If the cake Indian ink is used, after rubbing it in a saucer, a piece of thin whalebone should be used for filling the ruling-pen.

## General Directions.

Keep all instruments perfectly clean: do not leave ink to dry in the ruling-pen.

In using dividers avoid, as much as possible, making holes through the paper.

The paper should be firmly fixed to the drawing-board by a drawing-pin at each corner, well pressed down. Do not stick pins in the middle of the board, because the points of the dividers are liable to slip into them and make unsightly holes in the paper.

A pencil sharpened to what is called a "chisel-point" is generally used for drawing lines ; it has the advantage of retaining its point longer, but a nicely-pointed pencil is better for neat work, as it enables you to see the commencement and termination of a line more easily.


Fig. 1.
Always rule a line from left to right, and slope the pencil slightly towards the direction in which it is moving; if this is done, there is less chance of indenting the paper, which should always be avoided.

Having determined the extent of a line, always rub out the superfluous length ; this will prevent unnecessary complication.

Avoid using India-rubber more than is necessary, as it tends to injure the surface of the paper. After inking in a drawing; use stale bread in preference to India-rubber for cleaning it up.

The tee-square should be used for drawing horizontal lines only (Fig. I) ; the perpendicular lines should be drawn by the set-squares. If this is done, it is immaterial whether the edges of the drawing-board are at right angles, because it will only be necessary to use one of its edges.

For drawing parallel lines that are neither horizontal nor perpendicular, hold one set-square firmly pressed upon the paper and slide the other along its edge (Fig. 2). Geometrical drawing can be greatly facilitated by the proper use of set. squares, so it is advisable to practise their use.


Fig. 2.
When a problem contains many arcs of circles, it is advisable to connect each arc with its corresponding centre. Enclose the centre in a small circle ; draw a dotted line to the arc, terminated by an arrow-head (Fig. 3).

In drawing intersecting arcs for bisecting lines, etc., the arcs should not intersect each other too obtusely or too acutely: the nearer the angle between the arcs approaches $90^{\circ}$ the easier it will be to ascertain the exact point required.

In joining two points by a line, first place the point of the pencil on one point, then place the edge of the ruler against it, and adjust the ruler till its edge coincides with the other point.


Fig. 3.
All the problems should be drawn larger than shown.
Great accuracy is required in drawing the various problems. Every effort should be made to ensure neatness and precision in the work.

All arcs should be inked in first, as it is easier to join a line to an arc than an arc to a line.

## CHAP'TER I.

## GEOMETRICAL DEFINITIONS.

A point simply marks a position ; it is supposed to have no magnitude.

A line has length without breadth or thickness. The extremities and intersections of lines are points. A straight line or right line is one that is in the same direction throughout its length, and is the shortest that can be drawn between two points. To produce a line is to lengthen it.

A plane is a flat even surface; it has length and breadth only. The intersections of planes are straight lines.

Parallel lines are straight lines in the same plane, and at equal distances apart throughout their entire length ; if produced they would never meet (Fig. 4).

Fig. 4.
A circle is a plane figure bounded by a curved line, such that all straight lines drawn to it from a certain point are equal. This point is called the centre, and the curved line is called the circumference of the circle.

A straight line drawn from the centre to the circumference, as ce or $c d$ (Fig. 5) is called a radius. A straight line drawn through the centre, and terminated at both ends by the circumference, as $a \dot{b}$, is called a diameter. A semicircle is


Fig. 5. half a circle, as $a d b$. A quadrant is a quarter of a circle, as $a d c$.

An are is any portion of the circumference of a circle, as $a b c$ (Fig. 6). A chord is a straight line joining the extremities of an arc, as $a c$. A segment is the space enclosed by the arc and its chord, as $f$. A sector is the space enclosed by two radii and the arc between them, as $g$. A tangent is line touching the circumference, as de; it is always at right angles to the radius of the circle at the point of contact.


Fig. 6.

An ordinate is a line drawn from a point in a curve perpendicular to the diameter, as dotted line in Fig. 7.


Fig. 7.

An abscissa is the part of the diameter cut off by the ordinate, as dotted line in Fig. 8.


Fig. 8.
An angle is the inclination of two straight lines meeting in a point. This point is called the vertex of the angle, as $a$ (Fig. 9). The angle here shown would be called either bac or cab.


Fig. 9.

If two adjacent angles made by two straight lines at the point where they meet be equal, as $d c a$ and $d c b$ (Fig. 5), each of these angles is called a right angle, and either of the straight lines may be said to be perpendicular to the other.

A right angle is supposed to be divided into 90 equal parts, each of which is called a degree. A degree is expressed in writing by a small circle placed over the last figure of the numerals denoting the number of degrees-thus $36^{\circ}$ means thirty-six degrees.

The circumference of a circle is supposed to be divided into 360 equal arcs, each of which subtends an angle of $I^{\circ}$ at the centre. Sometimes this arc is itself loosely termed a degree.

An angle containing more than $90^{\circ}$ is called an obtuse angle, as ecb (Fig. 5) ; while an angle containing less than $90^{\circ}$ is called an acute angle, as ace.

A line is said to be perpendicular to a plane when it is at right angles to any straight line in that plane which meets it.

Concentric circles have the same centre.

## Triangles.

Triangies are closed figures contained by three straight lines.
A triangle which has all its sides equal is called equilateral (Fig. Io).
N.B.-Such a triangle will always have its three angles equal, and therefore will also be equiangular.


Fig. 10.

A triangle which has two sides (and therefore two angles) equal is called isosceles (Fig. II) (isos, equal ; skelos, a leg).


Fig. II.


Fig. 12.

A triangle which has an obtuse angle is called obtuse-angled (Fig. I3).


Fig. 13.

A triangle which has three acute angles is called acute-angled (Fig. 14).


Fig. 14.

The base of a triangle is its lowest side, as $a b$ (Fig. IO).
The vertex is the point opposite the base, as $c$ (Fig. Io).
The altitude or perpendicular height is a line drawn from the vertex at right angles to its base, as $c d$ (Fig. 10).

The median is a line drawn from the vertex to the middle point of the base.

## Quadrilateral Figures.

Quadrilateral figures are such as are bounded by four straight lines.

A quadrilateral figure whose opposite sides are parallel is called a parallelogram (Fig. I5). N.B. - The opposite sides and angles of parallelograms are equal.


Fig. 15.

A parallelogram whose angles are right angles is called a rectangle (Fig. 16) or oblong.

Fig. 16.

A rectangle which has its sides equal is called a square (Fig. 17).

Fig. 17.
A parallelogram whose angles are not right angles is called a rhombus, if its sides are all equal (Fig. I8), or a rhomboid if the opposite sides alone are equal-Gk. rhombos, from rhembein, to twirl, from some likeness to a spindle.

All other quadrilaterals are called trapeziums.
A line joining two opposite angles of a quadrilateral figure is called a diagonal, as the dotted line $a b$ (Fig. 15).

## Polygons.

A polygon is a plane figure which has more than four angles.
A polygon which is both equilateral and equiangular is called regular.

A polygon of five sides is called a pentagon.

| $"$ | $"$ | six | $"$ | $"$ |
| :--- | :--- | :--- | :--- | :--- |
| $"$ | hexagon. |  |  |  |
| $"$ | seven | $"$ | $"$ heptagon. |  |
| $"$ | eight | $"$ | $"$ an octagon, etc. |  |

## Solids.

A solid has length, breadth, and thickness. A solid bounded wholly by planes is called a polyhedron (poly, many ; hedra, a side).

A solid bounded by six planes or faces, whereof the opposite ones are parallel, is called a parallelepiped (parallelos, paralle] ; and epipedon, a plane).

A parallelepiped whose angles are all right angles is called a rectangular parallelepiped or orthohedron (orthos, right ; and hedra, a side).

An orthohedron with six equal faces is called a cube (kubos, a die).

A polyhedron, all but one of whose faces meet in a point, is called a pyramid (Gk. pyramis, a pyramid).

Pyramids are often named, after the shape of their bases, triangular, square, etc.

A polyhedron, all but two of whose faces are parallel to one straight line, is called a prism (Gk. prisma, from prizein, to saw, a portion sawn off).

If the ends of a prism are at right angles to the straight line to which the other faces are parallel it is called a right prism.

Prisms are often named, after the shape of their ends, triangular, hexagonal, etc.

A cylinder is a solid described by the revolution of a rectangle about one of its sides which remains fixed. This fixed line is called the axis of the cylinder.

A right circular cone-generally spoken of simply as a cone -is a solid described by the revolution of a right-angled triangle about one of the sides containing the right angle, which side remains fixed. This fixed line is called the axis of the cone ; the base is a circle, and the point opposite the base is called the vertex.

A solid bounded by a closed surface, such that all straight
lines drawn to it from a certain point are equal, is called a sphere (Gk. sphaira, a ball).

The point referred to is called the centre of the sphere.

## Technical Definitions.

A plane parallel to the ground, or, more strictly speaking, parallel to the surface of still water, is called a horizontal plane.

A vertical plane is a plane at right angles to a horizontal plane.

A horizontal line is a line parallel to a horizontal plane.
A vertical line is a line at right angles to a horizontal plane.


Fig. 19.
The plan of an object is the tracing made on a horizontal plane by the foot of a vertical line, which moves so as to pass successively through the various points and outlines of the object, as A and B (Fig. 19).

An elevation of an object is the tracing made on a vertical plane by the end of a horizontal line, at right angles to the vertical plane, which moves so as to pass successively through the various points and outlines of the object, as $\mathrm{A}^{\prime}$ and $\mathrm{B}^{\prime}$ (Fig. 19).
N.B.-A is an object parallel to the vertical plane, and B an object inclined to it.

## General Properties of some of the Figures already described.

If two lines cross each other the opposite angles are always equal. The angle $a c b$ is equal to the angle $d c e$, and the angle acd is equal to the angle bce (Fig. 20).

The two adjacent angles are equal to two right angles; the angles $a c d$ and $a c b$, for


Fig. 20. instance, as well as the angles bce and $c c d$.

Triangles. - The three angles of a triangle contain together $180^{\circ}$, or two right angles; so if two angles are given, the third angle can always be found. For example, if one angle is $70^{\circ}$ and the other $30^{\circ}$, the remaining angle must be $80^{\circ}$ (Fig. 21).

$$
\begin{array}{r}
70^{\circ}+30^{\circ}=100^{\circ} . \\
180^{\circ}-100^{\circ}=80^{\circ} .
\end{array}
$$

The exterior angle of a triangle is equal to the two opposite interior angles. The angle $a b c$ is equal to the angles $b c d$ and $c d b$ together; in the same way the angle $c d e$ is equal to the two angles $\bar{a}$ $b c d$ and $c b d$ (Fig. 22).


Fig. 21.


Fig. 22.

If we multiply the base by half the altitude, we get the area of a triangle; or half its base by its altitude will give us the same result.

Triangles of equal bases drawn between parallel lines are equal in area, and lines drawn parallel to their bases at equal heights are equal in length, as the dotted lines shown (Fig. 23).


Fig. 23.

If we bisect two sides of a triangle and join the points of bisection, we get a line that is always parallel to the third side (Fig. 24).


Fig. 24.
Quadrilaterals.-If we bisect the four sides of a quadrilateral figure and join the points, it will always give us a parallelogram, as shown by dotted lines. The reason for this will be apparent by the principle shown in the preceding figure if we draw a diagonal of the quadrilateral, so as to form two triangles (Fig. 25).

Parallelograms drawn between parallel lines on equal bases are always equal in area, and parallel lines drawn at equal heights are always equal to each other and to the bases, as shown by dotted lines (Fig. 26).


Fig. 26.

Semicircles.-Any two lines drawn from the extremities of the diameter, to a point on the circumference of a semicircle, will form a right angle.


Fig. 27.

## EXERCISES.

Note. -Feet are represented by one dash ('), and inches by two dashes (") ; 3 feet 6 inches would be written thus-- $3^{\prime} 6^{\prime \prime}$.

1. Draw lines of the following lengths: $3^{\prime \prime}, 4 \frac{1}{2}, 2 \frac{33^{\prime \prime}}{4}, 1 \frac{7^{\prime \prime}}{8}, 2.25^{\prime \prime}$, 3.50", 1.75".
2. Draw an acute angle, and an obtuse angle.
3. Draw the following triangles, viz. equilateral, scalene, isosceles, obtuse-angled, right-angled, and acute-angled.
4. Draw a right-angled triangle, and write the following names to its different parts, viz. hypotenuse, vertex, base, median, and altitude.
5. Draw the following figures, viz. rectangle, rhombus, square, rhomboid, trapezium, and parallelogram.
6. Draw a circle, and name the different parts, viz. sector, radius, chord, arc, diameter, segment, and tangent.

## CHAPTER II.

PROBLEMS ON LINES, TRIANGLES, QUADRILATERALS, CONVERGENT LINES, AND CIRCLES.

## Lines.

1. To bisect a given straight line AB.
From $A$ and $B$ as centres, with any radius greater than half the line, describe arcs cutting each other in C and D ; join CD. The straight line CD will bisect $A B$ in E. Also the line $C D$ will be perpendicular to AB.
2. To bisect a given arc AB .

Proceed in the same way as in Problem I, using the extremities of the arc as centres. The arc AB is bisected at E .


Fig. 29.
3. To draw a line parallel to a given line $A B$ through a given point C.


Fig. 30.

Take any point D in line AB , not opposite the point C ; with D as centre, and DC as radius, describe an arc cutting $A B$ in $E$, and from $C$ as centre, with the same radius, draw another arc DF ; set off the length EC on DF ; a line drawn through $C F$ will be parallel to $A B$.
4. To draw a line parallel to a given line AB at a given distance from it.


Let the length of the line $C$ represent the given distance. Take any points D and E in line $A B$ as centres, and with $C$ as radius describe


Fig. 3r. arcs as shown ; draw the line FG as a tangent to these arcs. FG will be parallel to AB.
5. From a point $C$ in a given line $A B$, to draw a line perpendicular to AB .


Fig. 32.

At the point $C$, with any radius, describe arcs cutting $A B$ in $D$ and $E$; with $D$ and $E$ as centres, and with any radius, draw arcs intersecting at $F$; join $F C$, which will be perpendicular to $A B$.
6. To draw a perpendicular to AB from a point at, or near, the end of the given line.

With B as centre, and with any radius BC , draw the arc CDE ; and with the same radius, starting at C , set off points D and E ; with each of these two points as centres and with any radius, draw arcs cutting each other at $F$; join FB , which will be perpendicular to AB .


Fig. 33.
7. To draw a line perpendicular to a given line, from a point which is without the line.

Let $A B$ be the given line and $C$ the point.

With C as centre, and with any radius greater than CD , draw arcs cutting the line AB in E and F ; from these points as centres, with any radius, describe arcs intersecting at G; join CG, which will be perpendicular to $A B$.


Fig. 34 .
8. To draw a perpendicular to $A B$ from a point opposite, or neariy opposite, to one end of the line.

Let $C$ be the given point. Take any point $D$ in $A B$, not opposite the point $C$; join CD and bisect it in E ; with E as centre, and ECas radius, draw the semi-circle CFD; join CF, which will be perpendicular to $A B$.

9. To divide a given line AB into any number of equal parts.

Take five for example.
 AB ; and from B , draw BC , parallel to AD. With any convenient radius, set off along AD , commencing at A , four parts (the number of parts, less one, into which it is required to divide the given line), repeat the same operation on $B C$, commencing at $B$, with same radius; join the points as shown, and the given line AB will be divided into five equal parts.

## 10. Another Method.



Fig. 37.

Draw AC at any angle to AB ; AC may be of any convenient length. With any radius, mark off along AC the number of equal parts required ; join the last division C with B ; draw lines from all the other points parallel to $C B$, till they meet $A B$, which will be divided as required.
11. From a given point $B$, in a given line $A B$, to construct an angle equal to a given angle $C$.


From point C of the given angle as centre, and with any radius, draw the arc EF : and with the same radius, with B as centre, draw the arc GH ; take the length of the arc EF, and set it off on GH ; draw the line BD through H . Then the angle GBH will be equal to the given angle $C$.
12. To bisect a given angle ABC .
From B as centre, and with any radius, draw the arc AC ; from A and C as centres, with any radius, draw the arcs intersecting at D ; join DB , which will bisect the angle $A B C$.

## 13. To trisect a right angle.

Let ABC be the right angle. From B as centre, and with any radius, draw the $\operatorname{arc} \mathrm{AC}$; with the same radius, and A and C as centres, set off points E and D ; join $E B$ and $D B$, which will trisect the right angle ABC .


Fig. 39 .


Fig. 40.
14. To trisect any angle $A B C$. ${ }^{1}$

From B, with any radius describe the arc AHC; bisect the angle $A B C$; join $A$ and $C$ cutting the bisector in D ; with D as centre, and with DA as radius, describe the semicircle AGC, cutting the bisector in G; and with the same radius, set off the points E and F from A and $C$; join $A G$; take the length $A G$ and set it off from $H$ along the line $G B$, which will give the point I ; join EI and FI, which will give the points J and K on the $\operatorname{arc} \mathrm{AHC}$;


Fig. 4 r. join J and K with B , which will trisect the angle ABC .
${ }^{1}$ This is one of the impossibilities of geometry ; but this problem, devised by the author, gives an approximation so near, that the difference is imperceptible in ordinary geometrical drawing.


Fig. 42.


Fig. 43.


Fig. 44.
16. On a given base AB to construct an isosceles triangle, the angle at veriex to be equal to given angle $C$.
Produce the base AB to E , and at A construct an angle FAE making with AE an angle equal to C. Bisect the angle FAB by the line AD. From B draw a line making with $A B$ an angle equal to DAB , and meeting AD in D . ADB will be the isosceles triangle required.
17. On a given base $A B$, to construct an isosceles triangle, its altitude to be equal to a given line CD.

Bisect AB at E , and erect a perpendicular EF equal in height to the given line $C D$; join $A F$ and $B F$, then $A F B$ will be the isosceles triangle required.
18. To construct a triangle, the three sides $A, B$, and $C$ being given.

Make the base DE equal to the given line $A$. From $D$ as centre, and with radius equal to line $B$, describe an arc at $F$, and from E as centre, and with radius equal to line $C$, draw another arc, cutting the other at F ; join FD and FE , which will give the triangle required.

$A \longrightarrow$


C Fig. 45 .
19. To construct a triangle with two sides equal to given lines $A$ and $B$, and the included angle equal to $C$.

Make an angle DEF equal to given angle $C$, in required position. Mark off EF equal to line A , and ED equal to line $B$; join DF. DEF is the triangle required.
20. To construct a triangle with a perpendicular height equal to $A B$, and the two sides forming the vertex equal to the given lines C and D .

Through B draw the line EF at right angles to AB . From A as centre, and with radii equal to the lengths of the lines $C$ and D respectively, draw arcs cutting the line EF; join AE and AF, which will give the triangle required.


Fig. 46.


Fig. 47.


Fig. 48.

$A \longrightarrow$
Fig. 49,


A
C—— Fig. 50
21. To construct a triangle, on the given base $A B$, with one base angle equal to $C$, and the difference of the sides equal to given line $D$.
At end A of the base, construct an angle equal to the given angle C. Cut off AF equal to line $D$, the given difference of the sides ; join FB. Bisect $F B$ at right angles by a line meeting AF produced in E ; join EB . AEB is the required triangle.
22. To construct a triangle on a base equal to given line A, with vertical angle equal to $D$, and sum of the twore. maining sides equal to BE .
Draw the line BE, and at E construct an angle making with BE an angle equal to half the given angle D . From point B, with radius equal to given line $A$, draw an arc cutting EC at C ; join BC. Bisect CE at right angles by line FG; join FC.

BCF will then be the triangle required.
23. To construct a triangle with two sides equal to the given lines $A$ and $B$ respectively, and the included median equal to given line C .
Draw a triangle with the side DE equal to given line $A$, the side $D F$ equal to
given line $B$, and the third side $F E$ equal to twice the given median C. Bisect the line FE at G. Join DG and produce $\mathrm{i}^{+}$ to H , making GH equal to DG ; join EH .

DEH will be the triangle required.
24. On a given base AB to describe a triangle similar to a given triangle DEF.
Make angles at A and B equal respectively to the angles at D and E. Produce the lines to meet at C . Then ABC will be the triangle required.


Fig. 5I

## Quadrilaterals.

25. To construct a square on a given base $A B$. DC and DB.

At point $A$ erect $A C$ perpendicular to AB and equal to it. With $B$ and $C$ as centres, and radius equal to AB , draw intersecting arcs meeting at $D$; join

CABD is the square required.


Fig. 52.
26. To construct a square on a given diagonal AB .


Fig. 53
27. To construct a rectangle with sides equal to given lines $A$ and $B$.


Draw the line $C D$ equal to given line A. At D erect a perpendicular DF equal to given line $B$. With $C$ as centre and radius equal to line $B$, and with $F$ as centre and radius equal to line $A$, draw arcs intersecting each other at E; join EC and EF, which will give the rectangle required.
28. To construct a rectangle with diagonal equal to given line A,


A
B. $\qquad$
Fig. 55. and one side equal to given line $B$.

Draw the line $C D$ equal to given line A. Bisect CD at E. With E as centre, and with radius EC, describe a circle ; from $C$ and $D$ as centres, and with given line $B$ as radius, set off the points $F$ and $G$; join $C G, G D$, DF, and FC.

CGDF is the required rectangle.
29. To construct a rhombus with sides equal to given line $A$, and angle

A.

Fig. 56.
equal to given angle $C$.
Make the base DE equal to given line A. At D construct an angle equal to given angle C. Set off DF equal to DE ; with F and E as centres, and radius equal to DE, draw arcs intersecting at $G$; join $F G$ and $E G$.

DEGF will be the required rhombus.
30. To draw a line bisecting the angle between two given converging lines AB and CD , when the angular point is inaccessible.

From any point E in AB , draw a line EF parallel to CD. Bisect the angle BEF by the line EG. At any point $H$ between $E$ and $B$, draw HL parallel to EG. Bisect EG and HL in M and N. Join MN, which, produced, is the bisecting line required.


Fig. 57.
31. Through the given point $A$, to draw a line which would, if produced, meet at the same point as the given lines $B C$ and DE produced.
Draw any convenient line FG; join AF and AG. Draw any line HK parallel to FG. At $H$ draw the line $H L$ parallel to $F A$, and at $K$ draw the line KL parallel to GA, cutting each other at L . Draw a line through L and A ; AL is the convergent line required.

## Circles.

32. To find the centre of a circle.
Draw any chord $A B$, and bisect it by a perpendicular DE, which will be a diameter of the circle. Bisect DE in C, which will be the centre of the circle.


Fig. 58.


Fig. 59.
33. To draw a circle through three given points $A, B, C$.


Fig. 60.

Join AB and BC. Bisect $A B$ and $B C$ by perpendculars cutting each other at D , which will be the centre of the circle. From D as centre, and DA as radius, describe a circle, which will then pass through the given points $\mathrm{A}, \mathrm{B}, \mathrm{C}$, as required.

Note. -The two following problems are constructed in the same manner.
34. To draw the are of a circle through three given points $\mathrm{A}, \mathrm{B}, \mathrm{C}$.
35. To find the centre of a circle from a given arc AC.
36. At the given equidistant points $A, B, C, D$, etc., on a given arc, to draw a number of radial lines, the centre of the circle being inaccessible.

With the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$,


Fig. 6r. D , etc., as centres, with radii larger than a division, describe arcs cutting each other at $E, F$, etc. Thus from $A$ and $C$ as centres, describe arcs cutting each other at E , and so on. Draw the lines $\mathrm{BE}, \mathrm{CF}$, etc., which will be the radial lines required.


Fig. 62.
37. To draw the arc of a circle through three given points $A, B, C$, the centre of the circle being inaccessible.
With A and C as centres, and with a radius equal to AC , draw indefinite arcs. From the points C and A draw lines through B till they meet
the arcs in D and E. From D and E set off short equal distances on the arcs above and below ; join the divisions on arc $A D$ to point $C$, and the divisions on arc $C E$ to point A. Where the lines from corresponding points intersect, we obtain a point in the arc ; for instance, where the line from EI intersects the line from $D_{1}$, we get the point $F$; and so on with the other points, by joining which with a fair curve, we get the required arc.

## EXERCISES.

1. Draw two parallel lines $2 \frac{3^{\prime \prime}}{4}$ long and $\frac{3^{\prime \prime}}{8}$ apart.
2. Draw a line $3.75^{\prime \prime}$ long ; at the right-hand end erect a perpendicular $2.25^{\prime \prime}$ high ; then, I. $50^{\prime \prime}$ from it, another perpendicular $\mathrm{I}_{\frac{7}{8 \prime \prime}}$ high ; and bisect the remaining length by a line $3^{\prime \prime}$ long, at right angles to it.
3. Draw a line $3 \frac{5^{\prime \prime}}{\mathrm{s}}$ long, and divide it into seven equal parts.
4. Draw a line $\mathbf{2}^{\frac{3}{4}}$ 少 long ; from the left-hand end mark off a distance equal to $1 \frac{1}{4}^{\prime \prime}$, and from the right-hand end a distance of $\frac{7^{\prime \prime}}{8^{\prime \prime}}$; draw another line $1.75^{\prime \prime}$ long, and divide it in the same proportion.
5. Mark the position of three points $\mathrm{A}, \mathrm{B}$, and $\mathrm{C}-\mathrm{A}$ to be $\mathrm{I}_{4}{ }^{\prime \prime}$ from B, B to be $2 \frac{1^{\prime \prime}}{}$ from $C$, and $C 1 \frac{7^{\prime \prime}}{8}$ from $A$; and join them.
6. Draw an angle equal to the angle ACB in the preceding question, and bisect it.
7. Draw a right angle, and trisect it ; on the same figure construct and mark the following angles, viz. $15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 75^{\circ}$; and also $7 \frac{1}{2}^{\circ}, 22 \frac{1}{2}^{\circ}$, and $37 \frac{1}{2}^{\circ}$.
8. Construct a triangle with a base $\mathrm{I}^{\frac{3}{4}}$ long; one angle at the base to be $60^{\circ}$, and the side opposite this angle $2^{\prime \prime}$ long.
9. Construct a triangle with a base $2.25^{\prime \prime}$ long, and altitude of 1. $75^{\prime \prime}$.
10. On a base $\mathrm{I}_{\frac{71}{\prime \prime}}$ long, construct an isosceles triangle ; the angle at its vertex to be $30^{\circ}$.
11. Draw a scalene triangle on a base $2^{\prime \prime}$ long; and construct a similar triangle on a base $1.75^{\prime \prime}$ long.
12. On a base $2 \frac{1^{\prime \prime}}{8}$ long draw a triangle with the angle at its vertex $90^{\circ}$.
13. Let a line $2.25^{\prime \prime}$ long represent the diagonal of a rectangle ; complete the figure, making its shorter sides $\frac{7{ }_{8}^{\prime \prime}}{}$ long.
14. Construct a rhombus with sides $1 \frac{3^{\prime \prime}}{}{ }^{\prime \prime}$ long, and one of its angles $60^{\circ}$.
15. Draw any two converging lines, and through any point between them draw another line which, if produced, would meet in the same point as the other two lines produced.
16. Fix the position of any three points not in the same line, and draw an arc of a circle through them.
17. Draw an arc of a circle, and on it mark the position of any three points; from these points, without using the centre, draw lines which, if produced, would meet in the centre of the circle containing the arc.
18. Construct a triangle, the perimeter to be $5.6^{\prime \prime}$, and its sides in the proportion of $5: 4: 3$.
19. Draw two lines $\mathrm{AB}, \mathrm{AC}$ containing an angle of $75^{\circ}$. Find a point $\mathrm{P}, \frac{3^{\prime \prime}}{4}$ from AB and $\frac{1_{2}^{\prime \prime}}{}{ }^{\prime \prime}$ from AC . Complete the isosceles triangle, of which BAC is the vertical angle, and the base passes through P.
(June, 'oo.)
20. Draw the figure shown (Fig. 63) according to the given figured dimensions.
(May, '97.)


Fig. 63.


Fig. 64.


Fig. 65.
21. Draw the pattern (Fig. 64) according to the given dimensions.
(April, '96.)
22. Copy the diagram (Fig. 65) enlarging it to the dimensions figured.
(April, '98.)
23. Draw the figure shown in the diagram, (Fig. 66) making the side of the outer hexagon $\frac{1}{4}^{\prime \prime}$ long.
(June, '98.)


Fig. 66.


Fig. 67.


Fig. 68.
24. Draw the corner ornament shown (Fig. 67), using the figured dimensions.
(April, '99.)
25. Draw the given diagram of window tracery (Fig. 68) using the figured dimensions. The arch is 'equilateral'.
(June, '99.)
26. Draw the given outline of window tracery (Fig. 69) using the figured dimensions. The arch is "equilateral," and all the arcs are of equal radius.
(April, 'oo.)


Fig. 69.


Fig. 70.
27. Draw the given frame (Fig. 70), using the figured dimensions. The border is $\frac{1_{4}^{\prime \prime}}{4}$ wide throughout. (April, 'oo.)
28. Draw the given figure (Fig. 71), using the figured dimensions. (This problem is intended as an exercise in the use of T and set squares.)
(June, 'oo.)


Fig. 71.


Fig. 72.
29. Draw the given figure (Fig. 72), using the figured dimensions. The spaces are $\frac{1_{4}^{\prime \prime}}{4}$ wide throughout.
30. Construct an isc,sceles right-angled triangle having its hypotenuse (or side opposite the right-angle) $2 \frac{3}{3}^{\prime \prime}$ long. Within it inscribe a square having one of its sides in the hypotenuse of the triangle. Measure and state, as accurately as you can, the length of one side of the square.
(June, 'ol.)

31. The given figure (Q. 31) is made up of a rectangle and semicircles. Make a copy of it, using the figured dimensions. (June, 'o2.)
22. Make a figure similar to the given figure (Q. 32) and having the height CD increased to $2 \frac{3^{\prime \prime}}{8}$. The centres of the arcs are given.
(June, 'o3.)

## CHAPTER III.

## POLYGONS.

Regular Polygons are figures that have equal sides and equai angles. To construct a regular polygon, we must have the length of one side and the number of


Fig. 73. sides; if it is to be inscribed in a circle, the number of its sides will determine their length.

If we take any polygon, regular or irregular, and produce all its sides in one direction only, Fig. 73, we shall find that the total of all the exterior angles, shown by the dotted curves, is equal to $360^{\circ}$, or four right angles ; and if we join each angle of the polygon to any point in its centre, the sum of the angles at this point will also be $360^{\circ}$, and there will be as many angles formed in the centre as there are exterior angles.
In regular polygons these angles at the centre will, of course, be equal to each other ; and if we produce the sides in one direction, as in Fig. 73, the exterior angles will be equal to each other ; and as the number of angles at the centre is equal to the number of exterior angles, and the sum of the angles in each instance is equal, the angle at the centre must equal the exterior angle.

To construct any regular polygon on the given line $A B$, for example, a nonagon,

$$
360^{\circ} \div 9=40^{\circ} .
$$

So if we draw a line at $B$, making an angle of $40^{\circ}$ with AB produced, it will give us the exterior angle of the nonagon, from which it will be


Fig. 74. easy to complete the polygon.

The perimeter of a polygon is sometimes given, e.g. Construct an octagon the perimeter of which is 6 inches.

$$
\begin{aligned}
& \text { inches. } \quad \text { inches. inches. } \\
& 6 \div 8=75=\frac{3}{4}
\end{aligned}
$$

Draw the line $A B$ this length. It has been shown that the exterior angles and those at the centre of a regular polygon are equal, $360^{\circ} \div 8=45^{\circ}$. Produce the line $A B$, and construct an angle of $45^{\circ}$. Make $\mathrm{BC}=\mathrm{AB}$. We now have three points from which we can draw the circle containing the required polygon (Prob. 33).

The polygon could also be


Fig. 75. drawn, after finding the length of AB , by any of the methods shown for constructing a polygon on a given straight line.

The centre of any regular polygon is the centre of the circle that circumscribes it.

Any regular polygon can be inscribed in a circle by making angles at the centre equal to the exterior angle as above.

If tangents to a circle circumscribing a regular polygon be drawn parallel to the sides of the inscribed polygon, or if tangents be drawn at the angles of the inscribed polygon, a similar figure will be described about the circle, and the circle will also be "described by," i.e. contained in, a similar figure.

In the following problems two general methods are given for cunstructing regular polygons on a given line, and two for
inscribing them in a given circle ; but as these general methods require either a line or an arc to be first divided into equal divisions, the special methods for individual polygons are preferable, which are also given.

See also how to construct any angle without a protractor (Prob. 134), and its application to polygons.
38. To inscribe in a circle, a triangle, square, pentagon, hexagon, octagon, decagon, or duodecagon.

Describe a circle, and draw the two diameters AE and BD at right angles to each other ; join $\mathrm{BA} . \mathrm{AB}$ is a side of a square.


Fig. 76. Set off on circumference, AF equal to $\mathrm{AC} ; \mathrm{AF}$ is a side of a hexagon ; join AF and EF ; EF is a side of an equilateral triangle. With D as centre, and radius equal to EF , mark off $G$ on EA produced. With $G$ as centre, and radius equal to AC , set off H on circumference; join AH ; AH is a side of an octagon. With $D$ and $E$ as centres, and radius equal to DC, set off the points I and J on circumference ; join ID ; ID is a side of a duodecagon. With CG as radius, and $J$ as centre, mark off $K$ on diameter BD. With CK as radius, mark off on circumference from $B$ the points L and M ; join BL and BM ; then BL is the side of a decagon, and $B M$ is a side of a.pentagon.

## Approximate Constructions, 39-42.

39. To inscribe any regular polygon in a given circle; for example, a heptagon.
Describe a circle, and draw the diameter $A B$. Divide $A B$
into as many equal divisions as there are sides to the polygon (in this instance seven). With A and B as centres, and with radius equal to $A B$, describe arcs intersecting at C . From C draw the line CD, passing through the second division from $\mathrm{A},{ }^{1}$ till it meets the circumference at $D$. Join AD, which will give one side of the polygon; to complete it, mark off AD round the circumference.

This method of constructing polygons is due to the Chevalier Antoine de Ville (1628), and


Fig. 77. although useful for practical purposes, is not mathematically correct.
40. To inscribe any regular polygon in a given circle (second method); for example, a nonagon.

Describe a circle, and draw the radius CA. At A draw a tangent to the circle. With A as centre, and with any radius, draw a semicircle, and divide it into as many equal parts as there are sides to the polygon (in this instance nine).

From point A draw lines through each of these divisions till they meet the circumference. Join these points, which will give the polygon required.


Fig. 78.
${ }^{1}$ Whatever number of sides the polygon may have, the line CD is always drawn through the second division from A.
41. On a given line $A B$, to describe a regular polygon; for example, a heptagon.
Produce $A B$ to $C$. With $A$ as centre, and $A B$ as radius,


Fig. 79. describe a semicircle, and divide it into as many equal divisions as there are sides to the polygon (in this instance seven). Join A with the second division from $C$ in the semicircle, ${ }^{1}$ which will give point D. Join AD. Through the points $D A B$ describe a circle. Set off the distance $A D$ round the circumference, and join the points marked, which will give the required polygon.
42. On a given line AB , to describe a regular polygon (second method) ; for example, a pentagon.

At point $B$ erect a perpendicular equal to $A B$. With $B$ as


Fig. 80. centre, and radius BA , draw the quadrant AC , and divide it into as many equal divisions as there are sides to the polygon (in this case five).

Join the point $B$ with the second division from C. ${ }^{1}$ Bisect AB at D , and erect a perpendicular till it meets the line drawn from $B$ to the second division, which will give point E. From E as centre, and with radius EB, describe a circle. From point $A$, with distance $A B$, mark off round the circum-

[^0]ference the points of the polygon. Join these points, which will give the pentagon required.
43. On a given straight line AB , to construct a regular pentagon. (True construction.)

At B erect BC perpendicular to $A B$, and equal to it. Bisect AB in D . With D as centre, and DC as radius, draw the arc CE meeting AB produced in E . With A and B as centres, and radius equal to AE , draw arcs intersecting at F. With A, B, and F as centres, and radius equal to AB , draw arcs intersecting at H and K . Join AH,


Fig. 8x. $\mathrm{HF}, \mathrm{FK}$, and KB. Then AHFKB will be the pentagon required.
44. On a given straight line AB , to construct a regular hexagon.

With A and B as centres, and radius AB , draw the arcs intersecting each other at C. With C as centre, and with the same radius, draw a circle. With the same radius commencing at $A$, set off round the circle the points $\mathrm{D}, \mathrm{E}, \mathrm{F}$, G. Join AD, DE, EF, FG, and GB, which will give the hexagon required.


Fig. 82.
45. In a given circle to inscribe a regular heptagon (approximately).


Draw any radius $A B$, and bisect it in C. Through C draw DE perpendicular to AB . With CD as radius, commencing at E , set off round the circle the points $F, G, H, K, L$, and $M$, by joining which we get the required heptagon.

Fig. 83.
46. On a given line AB , to construct a regular heptagon (approximately).
With $B$ as centre, and $B A$ as radius, draw a semicircle


Fig. 8 \&. meeting AB produced in C. With centre $A$, and radius AB , draw an arc cutting the semicircle in D. Draw DE perpendicular to AB . With C as centre, and radius equal to DE , draw an arc cutting the semicircle in F . Join BF. From the three points $A, B, F$, find the centre of the circle H (Prob. 33). With H as centre, and radius HA, draw a circle. With AB as radius commencing at F , set off on the circle the points $K, L, M$, and $N$, by joining which we get the heptagon required.
47. On a given line AB , to construct a regular octagon.

At A and B erect the perpendiculars AC and BD. Produce AB to E . Bisect the angle DBE by the line $B F$. Make $B F$ equal to AB . From F draw the line FH parallel to AB , and make KH equal to LF. Join AH. Set off on AC and BD , from points $A$ and $B$, a length equal to HF , which will give the points C and D . With the points C, D, H, and F as centres, and a radius equal to AB , draw


Fig. 85. arcs intersecting at M and N. Join HM, MC, CD, DN, and NF, which will give the octagon required.
48. In a given circle to inscribe a nonagon.

Draw the diameters AB and CD perpendicular to each other. With A as centre, and radius AE , draw the arc cutting the circle in F . With B as centre, and radius BF , draw the arc cutting CD produced in G. With G as centre, and radius GA, draw the arc cutting CD in H . With HC as radius, commencing at A , set off on the circle the points $\mathrm{K}, \mathrm{L}, \mathrm{M}$,


Fig. 86. $\mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}$, and R , by joining which we get the nonagon required.
49. In a given circle to inscribe a regular undecagon.

Draw the two diameters AB and CD cutting each other in E .


With A as centre, and AE as radius, draw an arc cutting the circle in H. With D as centre, and DE as radius, draw an arc cutting the circle in $F$. With $F$ as centre, and FH as radius, draw the arc cutting CD in K. Join HK. Then HK is equal to one side of the undecagon. With HK as radius, starting from H , set off cn the circle the points of the required undecagon, and join them.

## EXERCISES.

1. Inscribed in circles of varying diameters, draw the regular polygons, from a pentagon to a duodecagon, by a general method and figure the angles formed by their sides.
2. In circles of various radii, draw all the preceding polygons by special methods. Join their angles to the centre of the circle by radii, and figure the angles between the radii.

3 On lines varying in length, draw the same polygons by a general method.
4. Construct on lines of different lengths the same polygons by special methods.
5. Construct an irregular hexagon from the following data: Sides,
 $130^{\circ}, \mathrm{CDE} 110^{\circ}, \mathrm{DEF}_{120^{\circ}}$.
6. Construct an irregular pentagon from the following data: Sides, AB 1.25", BC 1.3", CD 1.2", DE 1.3", EA $1.4^{\prime \prime}$; Diagonals, AC 1. $8^{\prime \prime}$ AD 1. $6^{\prime \prime}$.
7. Construct a regular polygon with one side $1^{\prime \prime}$ in length and one angle $140^{\circ}$.
8. How many degrees are there in each of the angles at the centre of a nonagon ?
9. Construct a regular polygon on the chord of an arc of $72^{\circ}$.
10. Inscribe in any given circle an irregular heptagon whose angles at the centre are respectively $52^{\circ}, 73^{\circ}, 45^{\circ}, 63^{\circ}, 22^{\circ}, 36^{\circ}$, and $69^{\circ}$.
11. Construct a regular octagon of $I^{\prime \prime}$ side, and a second octagon having its angles at the middle points of the sides of the first.
(May, '97.)
12. Draw a figure similar to the one shown (Fig. 88), the points of the star being at the angles of a regular heptagon inscribed within a circle of $1^{\frac{1}{4}}{ }^{\prime \prime}$ radius.
(June, '97.)
-3. Within a circle of $\mathrm{I}_{2}^{1 \prime \prime}$ radius inscribe a regular nonagon. Within the nonagon inscribe a rectangle having all its angles in the sides of the nonagon, and one of its sides $I_{\frac{1}{2}}{ }^{\prime \prime}$ long.


Fig. 88.
14. Construct a regular pentagon of $i \frac{1}{2}$ " side. Describe five circles of $\frac{3^{\prime \prime}}{}{ }^{\prime \prime}$ radius, having their centres at the five angles of the pentagon.
(June, '98.)
15. Draw the given figure (Q. 15), making the radius of the outer circle $\mathrm{I}_{\frac{1}{4}}{ }^{\prime \prime}$.
(April, 'or.)
16. Construct a regular pertagon of $2^{\prime \prime}$ side, and a similar pentagon of $2^{\prime \prime}$ diagonal. The two figures should have the same centre.
(N.B. - The protractor may not be used for obtaining the angle of the pentagon.)
(June, 'o2.)

Q. 15 .

Q. 17.

Q. 19.
17. Copy the diagram (Q. 17), using the figured dimensions.
(June, 'o2.)
18. Within a circle of $\mathrm{I} \cdot 5^{\prime \prime}$ radius inscribe a regular nonagon. With the same centre describe a regular nonagon of $0.75^{\prime \prime}$ side. (May, 'o3.)
19. Copy the given figure (Q. 19). The straight lines are to form a regular pentagon of $1 \cdot 8^{\prime \prime}$ side, and the five equal segments are to have their arcs tangential at the angles of the pentagon. Show all your constructions clearly.
(June, 'o3.)

## CHAPTER IV.

## INSCRIBED AND DESCRIBED FIGURES.

50. To inscribe an equilateral triangle in a given circle ABC .

Find the centre E (Prob. 32), and draw the diameter DC.


Fig. 89. With D as centre, and DE as radius, mark off the points A and B on the circumference of the circle. Join $\mathrm{AB}, \mathrm{BC}$, and CA. Then ABC will be the inscribed equilateral triangle required.
51. To describe an equilateral triangle about a given circle ABC.
At the points $A, B$, and $C$ draw tangents to the circle (Prob. 84), and produce them till they meet in the points F, G, and


Fig. 90. $H$. Then the equilateral triangle FGH will be described about the circle ABC , as required.
52. To inscribe a circle in a given triangle ABC .

Bisect the angles CAB and ABC by lines meeting in D . From D let fall the line DE , perpendicular to AB . With D as centre, and DE as radius, inscribe the circle required.
53. To describe a circle about a given triangle ABC .
Bisect the two sides $A B$ and $A C$ perpendicularly by lines meeting in D. With D as centre, and DA as radius, describe the required circle.
54. To describe an equilateral triangle about a given square ABDC .
With C and D as centres, and with CA as radius, describe arcs cutting each other at E. With E as centre, and with the same radius, mark off the points $F$ and $G$ on these arcs. Join CF and DG, and produce them till they meet in the point H , and the base AB produced in K and L. Then HKL is the required equilateral triangle.
55. In a given triangle ABC , to inscribe an oblong having one of its sides equal to the given line D .
From A, along the base $A B$, set off $A E$ equal to the given line D. From E draw the line EF parallel to AC. Through F draw the line FG parallel to the base AB. From $G$ and $F$ draw the lines GH and FL perpendicular to AB . GFLH is the oblong required.


Fig. 9 r.


Fig. 92.


Fig. 93.


Draw any two diameters $A B$ and $C D$ at right angles to each other. Join the extremities of these diameters. ACBD is the inscribed square required.
57. To describe a square about a given circle.
At the points $\mathrm{A}, \mathrm{C}, \mathrm{B}, \mathrm{D}$ draw tangents meeting each other at the points E, F, G, H. (Prob. 84). EFHG is the described square required.
58. To inscribe a circle in a given square.


Fig. 95.


Fig. 96.

Draw the diagonals AB and CD intersecting each other at E. From E draw EF perpendicular to AD. With E as centre, and EF as radius, draw a circle. This will be the in scribed circle required.
59. To describe a circle about a given square.
With centre E , and radius EA, draw a circle. This will be the described circle required.
60. To inscribe a square in a given rhombus.
Draw the two diagonals $A B$ and CD. Bisect the angles AEC, AED, and produce the lines each way till they meet the sides of the rhombus in the points F, G, H, L. Join FG, GL, LH, and HF. FGLH is the inscribed square required.
61. To inscribe a circle in a given rhombus.
Draw the diagonals AB and $C D$ intersecting each other in E. From E draw the line EF peıpendicular to $A$ AD (Prob. 7). With E as centre, and EF as radius, draw a circle. This is the inscribed circle required.
62. To inscribe an equilateral triangle in a given square ABDC.
With B as centre, and radius BA , draw the quadrant AD ; and with same radius, with A and $D$ as centres, set off on the quadrant the points F and E. Bisect AE and FD, and through the points of bisection draw lines GB and $H B$, cutting the given square in G and H. Join GH. BGH is the inscribed equilateral triangle required.
63. To inscribe an isosceles triangle in a given square ABDC, having a base equal to the given line E .
Draw the diagonal AD. From A, along AD, set off AF equal to half the given base E. Through F draw the line GH perpendicular to AD (Prob 5). Join GD and HD. GDH is the inscribed isosceles triangle required.


Fig. 07.


Fig. 98.

$E$
Fig. 99.
64. To inscribe a square in a given trapezium ACBD which has its adjacent pairs of sides equal.


Fi:- 100

lig. rox.

Draw the two diagonals $A B$ and CD. From point C set off CE perpendicular and equal to CD. Join EA by a line cutting CB in G. Draw the line GF parallel to AB. From points F and G draw the lines FH and GK parallel to CD. Join HK. FGKH is the inscribed square required.
65. To inscribe a circle in a given trapezium ACBD which has its adjacent pairs of sides equal.
Bisect any two adjacent angles, as ADB and DBC , by lines meeting in E. From E draw EF perpendicular to CB. With E as centre, and EF as radius, draw a circle. This will be the inscribed circle required.
66. To insert a rhombus in a given rhomboid ABDC .


Draw the diagonals AD and $C B$ intersecting at E . Bisect two adjacent angles, as CED and DEB, by lines cutting the given rhomboid in $F$ and $G$, and produce these lines to K and H . Join FG, GK, KH, and HF. FGKH is the inscribed rhombus required.
67. To inscribe an octagon in a given square ABDC .

Draw the two diagonals AD and CB intersecting each other in E . With A as centre, and AE as radius, mark off the points F
and $G$ on the sides of the square. Proceed in the same manner with the angles $\mathrm{B}, \mathrm{C}$, and D as centres, which will give the eight points required on the given square, byjoining which we obtain the required inscribed octagon.
68. To inscribe a square in a given hexagon ABCDEF.
Draw the diagonal EB; bisect it in G, and draw HK perpendicular to it. Bisect two adjacent angles, as BGK and EGK, by lines meeting the hexagon in O and M . Produce these two lines till they meet the opposite sides of the hexagon in L and N . Join LM, MO, ON, and NL. LMON is the inscribed square required.
69. To inscribe four equal circles in a given square ABDC; each circle to touch two others, as well as two sides of the given square.
Draw the two diagonals AD and CB intersecting at E . Bisect the sides of the square in the points $\mathrm{F}, \mathrm{G}, \mathrm{H}$, and L (Prob. i). Draw FH and GL. Join FG, GH, HL, and LF, which will give the points M, $\mathrm{O}, \mathrm{P}$, and R. From M draw MN parallel to $A B$. With $\mathrm{M}, \mathrm{O}, \mathrm{P}$, and R as centres, and radius equal to MN , describe the four inscribed circles required.


Fig. 105.
70. To inscribe four equal circles in a given square ABDC ; each circle to touch two others, and one side only of the given square.


Draw the two diagonals AD and CB intersecting in the point E . Bisect the sides of the square in the points $F, G, H$, and L. Join FH and GL. Bisect the angle GDE by a line meeting GL in M. With centre E, and radius EM, mark off the points N, O, and P. With the points $\mathrm{M}, \mathrm{N}, \mathrm{O}$, and P as centres, and radius equal to MG, describe the four inscribed circles required.
71. To inscribe three equal circles in a given equilateral triangle ABC ; each circle to touch the other two, as well as two sides of the given triangle.


Fig. 107.

Bisect the two sides of the triangle at right angles by lines meeting at the centre D. Draw a line from point $C$ through the centre D till it meets the base at E. Bisect the angle $B E C$ by a line meeting DB in F. With D as centre, and DF as radius, mark off the points $K$ and L. From $F$ draw the line FH perpendicular to AB . With the points $\mathrm{F}, \mathrm{K}$, and L as centres, and with a radius equal to FH , draw the three inscribed circles, required.
72. In a given equilateral triangle $A B C$, to inscribe three equal circles touching each other and one side of the triangle only. Bisect two sides of the triangle at right angles by lines
meeting at the point D . Join CD and produce it to E . Bisect the angle DBE by a line cutting CE in F. With D as centre, and DF as radius, mark off the points $G$ and $H$. From the points $F, G$, and H as centres, and with a radius equal to FE draw the three inscribed circles required.
73. In a given equilateral triangle ABC , to inscribe six equal circles touching each other.

Having drawn the three circles according to the last


Fig. ros. problem, draw lines through G and H , parallel to the sides of the triangle, till they meet the lines bisecting the angles in the points L, M, and N. These points are the centres of the three circles which will complete the six inscribed circles required.
74. In a given octagon to inscribe four equal circles touching each other.

Draw the four diagonals meeting in C. Bisect the angle ABC by a line intersecting AC in D. With C as centre, and CD as radius, mark off the points $F$, $G$, and $H$. From $D$ draw the line DL perpendicular to KC . With D, F, G, and H as centres, and a radius equal to DL, draw the four inscribed circles required.


Fig. rog.
75. In a given circle to draw four equal circles touching each other.


Fig. 110.

Find centre of circle E (Prob. 32). Draw the two diameters AB and CD at right angles to each other. At A and D draw tangents to the circle to meet at the point F (Prob. 84). Join FE. Bisect the angle EFD by a line cutting CD in G. With $E$ as centre, and EG as radius, mark off the points H, K, and L. With G, H, K , and L as centres, and with a radius equal to GD, draw the four inscribed circles required.
76. In a given circle to inscribe any number of equal circles touching each other. For example, five.
Find the centre C (Prob. 32). Divide the circumference into five equal parts (Prob. 38), and draw the five radii to meet the


Fig. iri. circumference in the points M, N, O, P, and R. Bisect the angle MCN by a line meeting the circumference in A. Through A draw a line at right angles to CE till it meets the lines CM and CN produced in B and D. Bisect the angle DBC by a line to meet CA in E. With C as centre, and CE as radius, draw a circle, and bisect each arc on this circle, between the five radii, in the points $F, G, H$, and $L$. With $\mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}$, and L as centres, and a radius equal to EA , draw the five inscribed circles required.
77. About a given circle $A$, to describe six circles equal to it, touching each other as well as the given circle.
Find centre of circle A (Prob. 32). With $A$ as centre, and a radius equal to the diameter of the given circle, draw the circle BCDEFG. Draw the diameter BE. Take the diameter of the given circle and mark off from $E$ the points $D, C, B, G$, and F. With each of these points as a centre, and a radius equal to that of the given circle, draw the six circles required.


Fig. 112.

## EXERCISES.

1. Within a given circle inscribe a square, and about the same circle describe an equilateral triangle.
2. Construct a rhombus with sides $1 \frac{1}{2}^{\prime \prime}$ long, and its shorter diagonal $1.75^{\prime \prime}$; inscribe a circle within it, and let the circle circumscribe an equilateral triangle.
3. Construct a trapezium with two of its sides $1 \frac{3^{\prime \prime}}{4}$ and two $11^{\prime \prime}$ respectively, and with its longer diagonal $2 \frac{1}{2}^{\prime \prime}$; inscribe within it a square, and let the square circumscribe an equilateral triangle.
4. Draw a regular hexagon with $\mathrm{I}^{\prime \prime}$ sides and let it circumscribe a square ; inscribe a regular octagon within the square.
5. Draw any triangle, and describe a circle about it.
6. Construct a square of $2 \frac{1}{2}$ " sides, and in it inscribe an isosceles triangle with a $\mathbf{I}_{4}^{3^{\prime \prime}}$, base; inscribe within the triangle a rectangle, one side of which is $\mathbf{I}_{4}^{\frac{1}{4}}{ }^{\prime \prime}$.
7. Within a square of $1.75^{\prime \prime}$ sides, inscribe an isosceles triangle with angle at vertex $60^{\circ}$; inscribe a circle within the triangle.
8. Within a circle of any radius, inscribe a regular duodecagon, and let it circumscribe a hexagon.
9. Construct a rhomboid with sides $2^{\prime \prime}$ and $I^{1 \prime \prime}$, its contained angle to be $60^{\circ}$; inscribe within it a rhombus.
10. Within an equilateral triangle of $3^{\prime \prime}$ sides, inscribe a circle, and within it 3 equal circles.
11. Draw a circle of $I_{\frac{1}{2}}{ }^{\prime \prime}$ radius; inscribe within it an equilateral triangle; inscribe within the triangle three equal circles touching each other and each one side of the triangle only.
12. Construct a square of $2^{\prime \prime}$ sides, and let it circumscribe four equal circles; each circle to touch two others, as well as two sides of the square.
13. Construct a square with sides of $2.3^{\prime \prime}$, and inscribe four equal circles within it ; each circle to touch two others, as well as one side only of the square.
14. Within a triangle of $2.7^{\prime \prime}$ sides, inscribe six equal circles.
15. Within a circle of $\mathbf{1 .} 7^{\prime \prime}$ radius, inscribe seven equal circles.
16. Draw two concentric circles, and between them, six equal circles, to touch each other as well as the two concentric circles.
17. In a decagon, inscribe five equal circles.
18. Construct an equilateral triangle of $2 \frac{3^{\prime \prime}}{4}$ side. Bisect all three of its sides and join the points of bisection. Within each of the four equilateral triangles thus formed inscribe a circle. (April, '98.)
19. Draw an equilateral triangle, a scalene triangle, a right-angled triangle, and an oblong; a trapezium, and a regular polygon of eleven sides, each in a 2 -inch circle, and write the names to each.
(May, '96.)
20. Draw each of the following figures in a separate 2 -inch circle, an isosceles triangle, an obtuse-angled triangle, an acute-angled triangle, a square, a rhombus, and a rhomboid, and write the name to each.
(May, '97.)
21. Draw a square, an obiong, and a trapezium ; a pentagon, a hexagon, an octagon, and two parallel straight lines, each in a separate 2 -inch circle, and write the name to each. (April, '98.)
22. About a square of $I^{\prime \prime}$ side describe a triangle having one of its angles $60^{\circ}$ and another $70^{\circ}$.
(May, '97.)
23. Within a rhombus, sides $2 \frac{1}{2}^{\prime \prime}$, one angle $60^{\circ}$, inscribe an ellipse touching the sides of the rhombus at their middle points. (June, '97.)
24. Construct a rectangle $2 \frac{11^{\prime \prime}}{\frac{1}{2}} \times \mathrm{I}_{\frac{3}{4}}{ }^{\prime \prime}$. Within it inscribe two other rectangles, each similar to the first, concentric with it and having their longer sides $I_{\frac{1}{2}}{ }^{\prime \prime}$ and $\mathrm{I}^{\prime \prime}$ long respectively.
(April, '98.)
25. Within an equilateral triangle of $3^{\prime \prime}$ side inscribe three equal circles each touching the two others, and tzoo sides of the triangle.
(May, '97.)
26. Construct a square of $\mathrm{I}_{\frac{1}{2}}$ " side, and within it inscribe a rectangle having one of its angles on each side of the square and one of its sides $1^{\prime \prime}$ long.
(June, 97.)
27. Construct a triangle, sides $1 \frac{1}{2}^{\prime \prime}, 2^{\prime \prime}$, and $2 \frac{1}{2}{ }^{\prime \prime}$, and within it
inscribe an equilateral triangle having its three angles in the three sides of the first triangle.
(June, '98.)
28. Within a square of $1 \frac{3^{\prime \prime}}{4}$ side inscribe a regular octagon having all its angles in the sides of the square.
(April, '99.)
29. Construct a regular heptagon of $I^{\prime \prime}$ side, and within it inscribe an equilateral triangle.
(June, '99.)
30. Construct a quadrilateral ABCD from the following data:-

$$
\begin{aligned}
& \text { Sides }-A B=\mathrm{I}^{\frac{1}{2}}{ }^{\prime \prime}, B C=\mathrm{I}_{2}^{\prime \prime} \\
& \text { Angles- } \mathrm{ABC}=105^{\circ}, \mathrm{BAD}=75^{\circ}
\end{aligned}
$$

The four angles of the figure all lie in the circumference of a circle.
(April, '98.)
31. Within a circle of $\mathrm{I}^{4^{\prime \prime}}$ radius inscribe a regular pentagon. About the same circle describe another regular pentagon, having its sides parallel to those of the inscribed pentagon.
(April, '96.)
32. Within a circle of $22^{\prime \prime \prime}$ diameter, inscribe four equal circles each touching the given circle and two of the others.
(June, '98.)
33. Within a circle of $\mathrm{I}^{\frac{3^{\prime \prime}}{}}$ radius inscribe a regular heptagon. Draw a second similar heptagon, of which the longest diagonals are $2^{\prime \prime}$ long.
(April, '99.)
34. Within a circle of $1_{\frac{1}{2}}{ }^{\prime \prime}$ radius, inscribe a regular hexagon. Within the hexagon inscribe three equal circles tonching each other, and each touching two sides of the hexagon.
(June, '99.)
35. Construct a rhombus having its sides $2^{\prime \prime}$ long and one of its angles $75^{\circ}$. Within it inscribe two equal circles touching each other, and each touching two sides of the rhombus.
(April, '99.)
36. Within a circle of $2 \frac{3^{\prime \prime}}{4}$ diam. inscribe a regular pentagon. Draw also a second regular pentagon concentric with the first one, its sides being parallel to those of the first and $\mathrm{I}^{\frac{1}{4}}{ }^{\prime \prime}$ long.
(June, I900.)
37. The sides of a triangle are $I^{\prime \prime}, I^{\frac{1}{4}}{ }^{\prime \prime}$ and $I_{\frac{1}{2}}{ }^{\prime \prime}$ long. About this triangle describe a circle, and abnut the circle describe a triangle of the same shape as the given one. The points of contact must be found and clearly shown.
(April, 1900.)
38. Describe a circle touching both the given circles (Fig. 113) and passing through the point $P$ on the circumference of the larger one, three times the size of figure.

> (April, 1900.)
39. About a circle of $I^{\prime \prime}$ diam. describe
 six equal circles, each touching the given circle and two of the others. Then describe a circle touching and enclosing all six of the outer ones.
40. Make an enlarged copy of the given diagram (Fig. 114), using the figured dimensions.
(June, '98.)


Fig. 114.


Fig. 115.
41. Copy the diagram (Fig. II5) according to the given dimensions. Show clearly how the centre of the small circle is determined.
(June, '98.)
42. Within a regular hexagon of $\mathrm{I}^{1{ }^{\prime \prime}}$ side, inscribe a square having all its angles in the sides of the hexagon. Within the square inscribe four equal circles, each touching two of the others and two sides of the square.
(April, 'oı.)
43. About a circle of $0.75^{\prime \prime}$ radius describe an equilateral triangle. Describe three equal circles touching the given circle and having their centres at the angles of the triangle. Determine the points of contact.
(April, '02.)
44. Draw the given figure (Q. 44), making the side of the square $2 \frac{1}{4}$ " long. Show clearly how all the points of contact are determined.
(April, '02.)

Q. 44 .
45. About a circle or $0 \cdot 8^{\prime \prime}$ radius describe a rhombus having one of its angles $54^{\circ}$. Each side of the rhombus touches the circle. Determine the four points of contact.
(June, '02.)
46. Describe a semicircle of $\mathrm{I}^{\prime} 5^{\prime \prime}$ radius. Within it inscribe two circles, each touching the other, and also touching the circumference and diameter of the semicircle.
(June, 'o3.)

## CHAPTER V.

## FOILED FIGURES.

Foiled figures are constructed on the regular polygons, and are of two kinds : viz. those having tangential arcs, and those having adjacent diameters.

Problem 78 is an illustration of the former kind. The arcs simply touch, and if produced would not intersect each other. The angles of the polygons are the centres of the circles containing the arcs.

Problem 79 is an illustration of the latter kind. The sides of th nolygon form the diameters of the semicircles, the centre vach side being the centre of the circle containing the arc. If these arre were produced, they would intersect each other.
78. To construct a foiled figure about any regular polygon, having tangential arcs. For example, a hexagon.
The Hexafoil. - Bisect one side $A B$ in $C$. With each of the angular points as centres, and AC as radius, draw the six arcs, as required.


Fig. 116.
79. To construct a foiled figure about any regular polygon, having adjacent diameters. For example, a pentagon.


Fig. 117.

The Cinquefoil. - Bisect each side of the pentagon in the points $C, D, E, F$, and G. With each of these points as centres, and with a radius equal to CA, draw the five arcs required.

Note.-If these arcs are to have a stated radius, the length of the line $A B$, in each instance, will be twice the required radius.
80. In a given equilateral triangle ABC , to inscribe a trefoil.


Fig. 18 .

The Trefoil. - Bisect the angles $C A B$ and $A B C$ by lines produced till they meet in L , and the sides of the triangle in D and E. From C, through centre L, draw the line CF. Join DE, EF, and FD. From G draw GI perpendicular to AC. With G, H, and K as centres, and a radius equal to GI, draw the three arcs till they meet each other, which will form the trefoil required.
81. Within a given circle, to inscribe three equal semicircles having adjacent diameters.

Find centre of circle A (Prob. 32). Draw the diameter BC, and
the radius AD perpendicular to it (Prob. 5). Trisect the angle BAD in E and F (Prob. 13). Set off DG equal to DF. Join FA and GA by lines produced to L and H . Join EG by a line cutting FL in M. With A as centre, and AM as radius, set off the points N and O . Join MN, NO, and OM, which are the diameters of the required semicircles. With $R, P$, and S as centres, and a radius equal to RM, draw the three semicircles required.


Fig. 119.
82. In a given square ABDC , to inscribe four semicircles having adjacent diameters.

The Quatrefoil. - Draw the diagonals AD and CB . Bisect each side of the square in the points $\mathrm{E}, \mathrm{F}, \mathrm{G}$, and H , and join EF and GH . Bisect HD in K and FB in L , and join KL , cutting GH in N. With P as centre, and PN as radius, mark off the points $\mathrm{M}, \mathrm{O}$, and R , and join $\mathrm{NM}, \mathrm{MO}, \mathrm{OR}$, and $\mathrm{RN}_{\text {, }}$ which will form the diameters of the required semicircles. With S, T, U, and $V$ as centres, and with a radius


Fig. 120. equal to SN , draw the four semicircles required.
83. Within a given circle to inscribe any number of equal semi. circles having adjacent diameters. For example, seven.
The Heptafoil. - Find centre C (Prob. 32). Divide the circle


Fig. 12 r . into as many equal parts as the number of semicircles required (in this case seven), and from each of these points, A, $\mathrm{B}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}$, and H , draw diameters. From the point $A$ draw the tangent AM (Prob. 84), and bisect the angle CAM by a line cutting CK in N (Prob. 12). With C as centre, and CN as radius, mark off the points $\mathrm{O}, \mathrm{P}$, $\mathrm{R}, \mathrm{S}, \mathrm{T}$, and U. Join each of these points to form the polygon. From the centre of each side of the polygon, with a radius equal to half of one of its sides, draw the seven semicircles required.

## EXERCISES.

1. Construct an equilateral triangle of $\mathrm{I} \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ sides, and about it describe a trefoil having tangential arcs.
2. Construct a pentagon of $\frac{3 \prime \prime}{4}$ sides, and about it describe a cinquefoil having adjacent diameters.
3. Draw a pentagon of $\frac{1_{2}^{\prime \prime}}{}$ sides, and about it construct a cinquefoil having tangential arcs.
4. In a circle of $I_{\frac{1}{2}}{ }^{\prime \prime}$ diameter, draw nine equal semicircles having adjacent diameters.
5. In a circle of $\mathbf{I}^{\prime \prime}$ diameter, inscribe a quatrefoil having tangential arcs.
6. Construct a regular decagon in a circle of $\frac{3}{4}$ radius, and within it inscribe a cinquefuil.

Note.-Foiled figures can be inscribed in all the regular polygons that have an even number of sides, by first dividing them into trapezia and then proceeding by the method shown in Prob. 65 ; or they can be drawn in circles divided into any number of equal sectors, by Prob. Io6.

## CHAPTER VI.

## TANGENTS AND TANGENTIAL ARCS

84. To draw a tangent to a given circle at a given point A.
Find the centre of the $B$ circle C (Prob. 32). Join AC. From $A$ draw the line $A B$ perpendicular to AC, and produce it. Then AB is the tangent required.


Fig. 122
85. To draw a tangent to a given circle from a given point A outside it.
Find the centre $C$ (Prob. 32). Join AC, and bisect in B. From B as centre, and with BA as radius, describe a semicircle cutting the circumference in D. Join AD , and produce it. Then the line AD is the tangent required.


Fig. 123.


Fig. 124 .


D
Fig. 125.

86. To draw a tangent to the arc of a circle at a given point A without using the centre.
Draw the chord AB and bisect it in C). At C erect a perpendicular to AB cutting the arc in D . Join AD. Make the angle DAE equal to DAC. Then EA produced is the tangent required.
87. To draw a circle with radius equal to line $D$ to touch two straight lines forming a given angle ABC.
Bisect the angle ABC by the line BE. Draw the line FH parallel to BA (Prob. 4), and at a distance equal to given line D from it. Where FH intersects BE will be the centre of the circle. With E as centre, and D as radius, draw the circle required.
88. To draw tangents to a circle from a given point A, the centre of the circle not being known.
From point $A$ draw any three secants to the circle. as $\mathrm{CB}, \mathrm{DE}$, and GF. Join BE and $\mathrm{DC}, \mathrm{DG}$ and FE , by lines intersecting in the points H and K . Draw a line through H and K till
it meets the circumference in L and M . Join AL and AM , which will be the tangents required.
89. In a given angle $C A B$, to inscribe a circle which shall pass through a given point $D$.

Bisect the angle CAB, by the line AE (Prob. 62). Take any convenient point $F$ in $A E$, and from $F$ draw the line FG perpendicular to AC (Prob. 7). With F as centre, and FG as radius, draw a circle. Join DA, cutting the circle in H. From the given point D draw DK parallel to HF. With K as centre, and KD as radius, draw the required circle.


Fig. 127.
90. To draw a circle which shall pass through the given point A and touch a given line BC in D .

At the given point $D$ erect the line DE perpendicular to BC (Prob. 5), and join AD. At point $A$ construct an angle DAE equal to the angle ADE (Prob. II). AE intersects DE in E. With E as centre, and ED as radius, draw the required circle.


Fig. 128.
91. To draw a circle which shall pass through the two given points $A$ and $B$ and touch the given line CD.


Fig. 129.

Join the two given points AB , and produce the line till it meets the given line CD produced in E . On AE describe the semicircle EFA; at B draw BF perpendicular to AE. From E along the line $E D$, set off $E G$ equal to EF. Then G, B, A are three points in the required circle, which can be drawn as required (Prob. 33).
92. To draw four equal circles, with radius equal to given line E , to touch two given lines AB and CD , which are not parallel.


Bisect the two adjacent angles by the lines FG and HK. Draw the lines LM and NO parallel tc the given line $C D$, at a distance from it equal tc the given radius E (Prob. 4). Where these lines intersect the bisectors, we get the points $S, R, T$, and P. With the points R, S, $T$, and $P$ as centres, and with a radius equal to E , draw the four circles required.

Fig. 130.
93. To draw an inscribed and an escribed circle, tangential to three given straight lines, forming a triangle ABC.

NOTE.-An escribed circle is also called an excircle.
Bisect the angles BAC and ACB by lines intersecting in F. From F draw the line FH perpendicular to AD. With $F$ as centre, and FH as radius, draw the inscribed circle required. Bisect the angle BCD by a line cutting the line AG in E. Draw the line EK perpendicular to AL. With E as centre, and EK as radius, draw the escribed circle required.


Fig. 13x. 94. A principle of inscribed and escribed circles.

If a triangle $A B C$ be taken, and $\mathrm{AF}, \mathrm{BD}$, and CE be lines drawn from the three angles perpendicular to the opposite sides, they will all intersect at the point H . Join the points $\mathrm{D}, \mathrm{E}, \mathrm{F}$. This will form a triangle of which H is the "incentre," being the centre of the inscribed circle. The centres of the escribed circles will be the points $A, B$, and $C$. The radii of the circles are found by drawing lines from the centres perpendicular


Fig. 132. to the sides of the triangle produced (Prob. 7), as the line $B K$.
95. To draw two circles tangential to three given straight lines, two of which are parallel.


Fig. 133.

Let AB and CD be the two given parallel lines, and let the third line intersect them in E and F . Bisect the four angles $\mathrm{AEF}, \mathrm{BEF}, \mathrm{CFE}$, and DFE by lines meeting at H and L. From $H$ draw the line $H M$ perpendicular to CD (Prob. 7). With $H$ and $L$ as centres, and a radius equal to HM , draw the two required circles.
Note.-A line joining $H$ and $L$ will be parallel to the two given lines $A B$ and $C D$.
96. To draw two circles tangential to three given straight lines, none of which are parallel ; the third line to be drawn to cut the other two.

Let $\mathrm{AB}, \mathrm{CD}$, and EF be the three given lines. Bisect the four


Fig. 134. angles AFE, BFE, CEF, and DEF by lines meeting at H and L . From H and L draw the lines HM and LN perpendicular to CD (Prob. 7). With H as centre, and radius HM , draw a circle ; with L as centre, and LN as radius, draw the other circle required.

Note. - A line produced through the points H and L would be the bisector of the angle formed by producing the lines $A B$ and CD.
97. To draw DIRECT COMMON TANGENTS to two given circles of unequal radii.
Let AC be the radius of one circle, and BD of the other. Join the centres $A$ and $B$. From the centre A draw a circle with a radius $=\mathrm{AC}-\mathrm{BD}$. Bisect the line AB at E (Prob. I). From E , with radius EA , draw a circle cutting the circle FKG in the points F and G . Join FB and GB. From F and B


Fig. 135 . draw the lines FO and BR perpendicular to FB (Prob. 7) ; and from the points $G$ and $B$ draw the lines $G P$ and $B S$ perpendicular to GB. Draw the line HL through the points O and R , and the line MN through the points P and S ; these will be the tangents required.
98. To draw TRANSVERSE COMMON TANGENTS to two given circles of unequal radii.

Let A and B be the centres of the given circles, and AC and BD their radii. Join AB . With $A$ as centre, and a radius $=\mathrm{AC}+\mathrm{BD}$, draw a circle. Bisect the line AB in E (Prob. 1). With E as centre, and a radius equal to EA, draw a circle cutting the circle FKG in the points F and G. Join AF and AG, cutting the given circle PCO


Fig. 136. in O and P . Join FB and GB . From B draw the line BS perpendicular to FB (Prob. 7), also the line BR perpendicular to GB. Draw the line HL through the points $O$ and $S$, and the line MN through the points $P$ and $R$; these will be the tangents required.

## Tangential Circles and Arcs.

99. Showing the principle of tangential circles.

One circle can touch another circle either internally or externally, and any number of circles can be drawn to touch a


Fig. 137. given line, as well as each other, in the same point. For instance take the point C on the given line AB . All circles that touch a given line in the same point touch each other at that point ; and all their centres will be on a line perpendicular to the given line.

If they are on the opposite sides of the given line, they will touch externally; and if on the same side, will touch internally. If they are on the same side of the line, one circle will be entirely within the other.
If their centres $F$ and $E$ are on the same side of the given line $A B$, the distance between them is equal to the difference of their radii ; but if their centres $E$ and $D$ are on the opposite sides of the given line, the distance between them is equal to the


D sum of their radii.

The point of contact can always be found by joining their centres.
100. To draw four equal circles with radius equal to given line $D$, with their centres on a given line $A B$; two to touch externally and two internally a given circle, whose centre is C and radius CG .
With centre $C$, and Fig. 138.
radius equal to the sum of the radii, i.e. $\mathrm{CG}+\mathrm{D}$, draw a circle cutting the given line AB in H and N . With centre C , and radius equal to the difference of the radii, i.e. $\mathrm{CG}-\mathrm{D}$, draw a circle cutting the given line $A B$ in $L$ and $M$. With $H, L, M$, and N as centres, and radius equal to D , draw the four circles required.
101. To draw four equal circles, with radius equal to given line D , with their centres on a given arc AB ; two to touch externally and two internally a given circle, whose centre is C and radius CG .

The construction of this problem is word for word the same as the last, the only difference being the words given arc instead of given line.


Fig. 139.

102 To describe a circle tan. gential to and including two given equal circles $A$ and $B$, and touching one of them in a given point $C$.
Find the centres of the two given circles $D$ and $E$, and join them (Prob. 32). Join CD. From C draw the line CK parallel to DE, meeting the given circle B in K . Join KE , and produce it till it meets $C D$ produced in $F$. With F as centre, and radius FC , draw the required circle.


Fig. 40.
103. To describe a circle tangential to and including two unequal given circles $A$ and $B$,
 and touching one of them in a given point $C$.
Find the centres D and E. Join CE. Cut off from $\mathrm{CE}, \mathrm{CH}$ equal in length to the radius of the smaller circle. Join DH. Produce CE. At D construct the angle HDF equal to the angle DHF (Prob. in). DF meets the line CE produced in $F$. With $F$ as centre, and FC as radius, draw the circle required.
104. To draw the arc of a circle having a radius of $1 \frac{1}{4}$ inches, which


Fig. 142. shall be tangential to two given unequal circles $A$ and $B$ and include them.
Note.-The diameters and distance between the circles must not be greater than $2 \frac{1}{2}$ inches.

Find the centres $D$ and $E$ (Prob. 32), and produce a line through them indefinitely in both directions, cutting the circles in K and L. From the points K and L on this line, set off KF and LH equal to the radius of the required arc, viz. $1 \frac{1}{4}$ inches. With D as centre, and a radius equal to DF , draw an arc at M ; and with E as centre, and EH as radius, draw an arc intersecting the other arc at M. From M draw the line MD, and produce it till it meets the circumference of the
larger circle in C . With M as centre, and MC as radius, draw the required arc.
105. To inscribe in a segment of a circle, whose centre is E, two given equal circles with a radius equal to line $D$.
From any radius EF cut off $\mathrm{FL}=\mathrm{D}$, and describe a circle with radius EL. Draw the line KL parallel to the base of the segment, and at a distance equal to given radius D from it (Prob. 4). Join EL and produce it till it meets the circumference in $F$. With $L$ and $K$ as centres, and LF as radius, draw the two required circles.


Fig. 143 .
106. In a given sector of a circle $A B C$, to inscribe a circle tangential to it.
Bisect the angle ACB by the line CD (Prob. I2). At D draw a tangent HL (Prob. 84) to meet the sides of the sector produced. Bisect the angle CLH by a line cutting $C D$ in $E$. With E as centre, and ED as radius, draw the circle required.


Fig. 144 .
107. Draw a circle having a radius of $\frac{1}{4}$ of an inch tangential to two given unequal circles A and B externally.
NOTE.-The circles must not be more than $\frac{1}{2}$ inch apart.

Find the centres $D$ and $E$ of the given circles (Prob. 32). From centre D , with the sum of the radii, i.e. $\mathrm{DK}+\frac{1}{4}$ of an inch, draw an arc at H ; and from centre E, with the sum of the other radii, i.e. $\mathrm{EL}+\frac{1}{4}$ of an inch, draw another arcat H . With H as centre, and radius HK , draw the circle required.


Fig. 145 .
108. To draw the arc of a circle tangential to two given unequal


Fig. 146. circles A and Bexternally, and touching one of them in a given point C .
Find the centres $D$ and $E$ of the given circles (Prob. 32). Join CE, and produce it indefinitely. Set off from C, on CE produced, CH equal to the radius of the larger given circle. Join DH. At D construct an angle HDF, equal to the angle FHD, to meet EC produced in F . With F as centre, and radius FC , draw the arc required.
109. To draw a circle, with a radius equal to given line $C$, tangential to two given unequal circles $A$ and $B$,


Fig. 147. to touch A externally and B internally.
Note.-The given radius must be greater than half the diameter of the enclosed circle and the distance between the circles.

Find the centres D and E of the two given circles (Prob. 32). From centre D , with radius, the sum i.e. $\mathrm{DH}+\mathrm{C}$, describe an arc at F ; and with E as centre and radius the difference, i.e. C-EK, draw another arc cutting the other at point F . Join FD and FE . With F as centre, and FH as radius, draw the circle required.
110. To draw a circle of $\frac{3}{8}$ of an inch radius tangential to the given line $A B$ and the given circle $C D E$.


Note.-The circle must be less than $\frac{3}{4}$ inch from the line. Draw the line KL parallel to AB , and $\frac{3}{8}$ of an inch from it. Find the centre $F$ of the given circle (Prob. 32), and with the sum-radius of the two circles draw the $\operatorname{arc} H M$ cutting KL in M. Join FM. With M as centre, and MN as radius, draw the required circle.

Fig. 148.

## EXERCISES.

1. Draw a circle $1.7^{\prime \prime}$ in diameter; at any point in its circumference, draw a tangent.
2. Draw a circle $1.25^{\prime \prime}$ in radius; from a point one inch outside the circle, draw a tangent to it.
3. Draw a circle $1^{\frac{7}{8}}{ }^{\prime \prime}$ in diameter ; from any point outside the circle, draw two tangents without using the centre.
4. Draw two lines, enclosing an angle of $45^{\circ}$; draw a circle $I \frac{11^{\prime \prime}}{}$ in diameter, tangential to these lines.
5. At any point in the arc of a circle draw a tangent, without using the centre.
6. Draw two circles of $2^{\prime \prime}$ and $\mathrm{I}^{\prime \prime}$ radii, with their centres $3^{\prime \prime}$ apart ; draw transverse common tangents to them.
7. Draw two circles of $1.7^{\prime \prime}$ and $\mathrm{I}^{\prime \prime}$ radii, with their centres $2.75^{\prime \prime}$ apart ; draw direct common tangents to them.
8. Draw two lines at an angle of $30^{\circ}$ with each other, and a third line cutting them both at any convenient angle; draw two circles tangential to all the three lines.
9. Construct a triangle with sides $2.25^{\prime \prime}$, I. $\mathrm{S}^{\prime \prime}$, and $\mathrm{I} .25^{\prime \prime}$; draw an inscribed and three escribed circles cangential to the lines forming the sides.
10. Draw two lines enclosing an angle of $45^{\circ}$; fix a point in any convenient position between these two lines, and draw a circle that shall pass through this point and be tangential to the two lines.
11. Draw a circle $\mathbf{1} .25^{\prime \prime}$ in diameter, and half an inch from it draw a straight line; draw a circle of $\frac{3^{\prime \prime}}{3^{\prime \prime}}$ radius, tangential to both the circle and the line.
12. Draw two circles of $1^{\prime \prime}$ and $\frac{1^{\prime \prime}}{2^{\prime \prime}}$ radius, with their centres $2 \frac{1^{\prime \prime}}{}$ apart ; draw another circle tangential to both externally.
13. Draw two circles $1.50^{\prime \prime}$ and $1^{\prime \prime}$ in diameter, their centres to be $2.25^{\prime \prime}$ apart ; draw another circle $3 \frac{1}{2}^{\prime \prime}$ in diameter, touching the larger circle externally, and the smaller one internally.
14. Draw two lines $\mathrm{AB}, \mathrm{AC}$, making an angle of $25^{\circ}$ at A . Describe a circle of $\frac{3 / 7}{4}$ radius touching $A B$ and having its centre on $A C$. From A draw a second tangent to the circle, marking clearly the point of contact.
(April, '96.)
15. Draw a line $\mathrm{AB}, 2^{\prime \prime}$ long. Describe a circle of $\frac{3^{\prime \prime}}{4}$ radius touching AB at A , and another of $\mathrm{I}^{\prime \prime}$ radius touching AB at B . Draw a second line which shall touch both circles, showing clearly the points of contact.
(June, '97.)
16. Describe two circles, each of $\frac{1}{2}^{\prime \prime}$ radius, touching each other at a point A. Find a point B on one of the circles, $\frac{7 \prime}{8}$ from A. Describe a third circle, touching both the others and passing through B. (April, '98.)
17. Construct a square of $2^{\prime \prime}$ side. In the centre of the square place a circle of $\frac{1^{\prime \prime}}{2}$ radius. Then describe four other circles, each touching the first circle and two sides of the square.
(June, 'oo.)
18. Draw the "cyma recta" moulding shown, (Fig. 149), adhering to the given dimensions. The curve is composed of two quarter-circles of equal radii, tangential to one another and to the lines AB and CD respectively.
(April, '96.)


Fig. 149.


Fig. 150.


Fig. 151.
19. Draw the "Scotia" moulding shown (Fig. 150). The curve is made up of two quarter-circles of $\mathrm{I}^{\prime \prime}$ and $\frac{1^{\prime \prime}}{}$ radius respectively.
(May, '97.)
20. Draw the "rosette" shown (Fig. 151), according to the figured dimensions.
(June, '97).
21. Draw the "ogee" arch shown (Fig. 152) to a scale of $\mathbf{2}^{\prime}$ to $\mathrm{I}^{\prime \prime}$. The arcs are all of $2^{\prime}$ radius. The methods of finding the centres and points of contact must be clearly shown.
(April, '99.)


Fig. 152.


Fig. 153.


Fig. ${ }^{54}$.
22. Draw the moulding shown (Fig. 153), adhering strictly to the figured dimensions. The arc of $\frac{1}{2}$ " radius is a quadrant. (April, '98.)
23. Draw the "cyma recta" moulding shown in the diagram (Fig. ${ }^{154}$ ), using the figured dimensions. The curve is composed of two
 equal tangential arcs each of $\frac{3^{\prime \prime}}{4}$ radius.
(June, '98.)
24. Describe a circle touching the given circle at T (Fig. 155) and passing through the point P (twice size of figure).
25. Describe the four given circles, using the figured radii (Fig. 156). The necessary construction lines and points of contact must be clearly shown.
(June, 'oo.)
26. Draw the "three centred" arch shown (Fig. 157). The two lower arcs are of $1^{\prime \prime}$ radius, and the upper arc is of $2 \frac{1^{\prime \prime}}{\prime \prime}$ radius. (April, 'oo.)


Fig. ${ }^{56}$.


Fig. 157.


Fig. 158.
27. Draw the "four centred" arch shown (Fig. 158), adhering to the figured dimensions.
(June, '97.)
28. Draw the figure shown, (Fig. I 59), making the sides of the square $2 \frac{11^{\prime \prime}}{}$ long.
(April, '96.)


Fig. 159.


Fig. 160.


Fig. 161.
29. Draw the figure shown (Fig. 160), adhering strictly to the figured dimensions.
(May,'97.)
30. Draw the figure shown (Fig. 161), according to the figured dimensions.
(June, '97.)
31. Draw the given figure (Fig. 162) making the side of the square $2 \frac{11 \prime}{4 \prime}$ and the radii of all the arcs $\frac{3^{\prime \prime}}{8}$.
(April, '99.)


Fig. 162.


Fig. 163.


Fig. 164.
32. Draw the figure shown (Fig. 163), using the figured dimensions. (April, '98.)
33. Copy the given figure (Fig. 164), using the figured dimensions.

## CHAPTER VII.

## PROPORTIONAL LINES.

Proportional lines may be illustrated by the example of simple proportion in arithmetic, in which we have four terms, e.g. 2:4::5:10

The relationship or ratio between the first two terms with regard to magnitude is the same as that between the second two. e.g. as 2 is to 4 , so is 5 to 10 ; therefore these four numbers are said to be in proportion.


Fig. 165.

The first and fourth terms are called the extremes, and the second and third the means.

The product of the extremes equals the product of the means, e.g. $2 \times 10=4 \times 5$. So, the first three terms being given, we can find the fourth. If we divide the product of the means by the first extreme we get the fourth proportional, e.g. $\frac{4 \times 5}{2}=10$.

Almost all geometrical questions on proportion are based on the following theorems :-

Take any triangle ABC, and draw any line DE parallel to one side, then-

$$
\mathrm{CD}: \mathrm{DA}:: \mathrm{CE}: \mathrm{EB}
$$

CD : CE : : CA : CB
CE : ED : : CB : BA
CD : DE : : CA : AB.
There are five varieties of proportional lines, viz.-
Greater fourth proportional.
Less fourth proportional.
Greater third proportional.
Less third proportional.
Mean proportional.

If the quantities be so arranged that the second term is greater than the first,-as $4: 6:: 8: x$, -the last term is called the greater fourth proportional.

If the terms are arranged so that the second term is less than the one preceding,-as $8: 6:: 4: x$,-the last or unknown term is called the less fourth proportional.

When the two means are represented by the same number,thus $4: 6:: 6: x$,-the answer or $x$ is called the third proportional.
The third proportional is found by dividing the square of the second by the first, e.g.-

$$
\frac{6^{2}}{4} \text { or } \frac{6 \times 6}{4}=9 .
$$

If the terms are placed so that the larger number is repeated, -thus $4: 6:: 6: x$, - the last term is called the greater third proportional; but if the terms are arranged so that the smaller number is repeated,-as $6: 4:: 4: x$,-the result is called the less third proportional.

The mean proportional between any two numbers is found by extracting the square root of their product, - e.g. $4 \times 9=36$; the square root of $36=6$, which is the mean proportional.
111. To find a fourth proportional to three given lines $\mathrm{A}, \mathrm{B}$, and C. THE GREATER FOURTH PROPORTIONAL.

Draw EH equal to given line $C$, and $E F$, at any angle with it, equal to given line B . Join FH , and produce EH to D , making ED equal to given line A. Draw DK parallel to FH till it meets EF produced in K (Prob. 3). Then KE will be the greater fourth proportional to the lines $\mathrm{A}, \mathrm{B}$, and C , i.e. $\mathrm{C}: \mathrm{B}:: \mathrm{A}: \mathrm{KE}$, e.g.

$\qquad$
$\qquad$
C if $\mathrm{C}=6$ feet, $\mathrm{B}=8$ feet, $\mathrm{A}=12$ feet, then $\mathrm{KE}=16$ feet.
112. To find a fourth proportional to three given lines $\mathrm{A}, \mathrm{B}$, and C .

THE LESS FOURTH PROPORTIONAL.


Draw the line DE equal to given line $A$, and EF , at any angle with it, equal to given line B. Join FD. From E , along ED, set off EG equal to given line C. From G draw GH parallel to FD (Prob. 3). Then HE is the less fourth proportional to the given lines $\mathrm{A}, \mathrm{B}$, and C , i.e. $\mathrm{A}: \mathrm{B}: \mathrm{C}: \mathrm{HE}$.
113. To find a third proportional between two given lines $A$ and $B$. THE GREATER THIRD PROPORTIONAL.


Draw CD equal to given line $A$, and CE, at any angle with it, equal to given line $B$. Join DE. With C as centre, and CD as radius, draw the arc DG to meet CE produced in G. From $G$ draw the line GF parallel to DE till it meets CD produced in F (Prob. 3). Then CF is the greater third proportional to the given lines A and B , i.e. $\mathrm{B}: \mathrm{A}:: \mathrm{A}: \mathrm{CF}$.
114. To find a third proportional between two given lines A and B .

THE LESS THIRD PROPORTIONAL.
Draw $C D$ equal to the given line $A$, and $C E$, at any
angle with it, equal to given line $B$. Join $D E$. From C as centre, and with radius CE, draw the arc EF cutting CD in F. Draw FG parallel tc DE (Prob. 3). Then CG is the less third proportional
 to the given lines A and B , i.e. $\mathrm{A}: \mathrm{B}:: \mathrm{B}: \mathrm{CG}$.

115. To find the MEAN PROPORTIONAL between two given lines $A B$ and $C D$.
Produce the given line $A B$ to E , making AE equal to the given line CD. Bisect the line EB in H (Prob. r ). From H as centre, and with radius HB , draw the semicircle EKB. At A draw the line $A K$ perpendicular to $E B$, cutting the semicircle in K (Prob. 5). Then AK is the mean provortional to the given lines $A B$ and $C D$, e.g. if $\mathrm{AB}=9$ feet and $\mathrm{CD}=4$ feet, then $\mathrm{AK}=6$ feet.
116. To divide a line in medial section, i.e. into EXTREME and MEAN proportion.
Let AB be the line. At A draw $A C$ perpendicular to AB , and equal to it. Bisect AC in D. With D as centre, and DB as radius, draw the arc cutting CA produced in E. With A as centre, and AE as radius, draw the arc cutting AB in F . Then $\mathrm{AB}: \mathrm{AF}:: \mathrm{AF}: \mathrm{BF}$.


Fig. 171.
117. To divide any straight line $A B$ in the point $C$, so that

$$
\mathrm{AC}: \mathrm{CB}:: 3: 4 .
$$



Fig. ${ }^{7}{ }^{2}$.

At A draw the line $A D$ of indefinite length, and at any angle to AB . From A , along $A D$, mark off seven equal distances of any convenient length. Join 7 B . At 3 draw the line ${ }_{3} C$ parallel to $7 B$ (Prob. 3). Then
$\mathrm{AC}: \mathrm{CB}:: 3: 4$,
$\mathrm{AB}: \mathrm{AC}:: 7: 3$.
118. To divide a line proportionally to a given divided line.

Let $A B$ be a given divided line, and GH a line which is


Fig. 173. to be divided proportionally to AB. Join GA and HB, and produce them to meet in C ; join C to the divisions in AB and produce them. These lines will divide GH as required. Divide BC into four equal parts, and draw the lines $\mathrm{D}, \mathrm{E}$, and F parallel to $A B$; join the divisions on $A B$ with $C$, then the divisions on the lines $\mathrm{D}, \mathrm{E}$, and F will represent respectively the proportions of $\frac{3}{4}, \frac{1}{2}$, and $\frac{1}{4}$ of the divisions on the given line $A B$.
119. To construct a triangle on a given line $A B$, so that the three angles may be in the proportion of $2: 3: 4$.

From B, with any radius, describe a semicircle and divide it into nine equal parts (Prob. 48). Draw the lines $\mathrm{B}_{4}$ and B 7 . Then the three angles 7 BC , 4 B 7 , and 4 BA are in the proportion of $2: 3: 4$. The sum of the three angles are equal to two right angles, because a semicircle contains $180^{\circ}$; so they must be the three angles of a triangle, because the three angles of any triangle are


Fig. ${ }_{174}$. together equal to two right angles. From A draw the line AD parallel to B 7 till it meets B 4 produced in D (Prob. 3). Then $A B D$ is the triangle required.
120. This problem illustrates an important principle in proportion.

Take a triangle ABC , the sides of which shall bear a certain ratio. For example, let $A B: B C$ as 2 : 1 . Produce $A B$ to $D$, and bisect the angles $A B C$ and $D B C$ by lines meeting $A C$ and $A C$ produced in H and E . Bisect the line HE in K (Prob. I). With K as centre, and KE as radius, draw the circle EBH. Now, if we take any point M in this circle, and join MA and MC, we shall find that they bear the same ratio as the lines


Fig. 175. AB and BC . In the example given MA: MC as $2: 1$. The same result would be obtained from any point in the circle.
121. To divide a right angle into five equal parts.


Fig. 176.

Let ABC be the right angle. Divide BC in D , so that $\mathrm{BC}: \mathrm{BD}:: \mathrm{BD}: \mathrm{DC}$ (Prob. in6). With C as centre, and CB as radius, describe the arc BE ; and with $B$ as centre, and $B D$ as radius, describe the quadrant DF, cutting BE in $\mathrm{E} \quad \mathrm{FE}$ is one-fifth of the quadrant FD. Arcs equal to it set off on FD will divide it into five equal parts.

122
To find the Arithmetic, the Geometric, and the Harmonic means between two given lines $A B$ and $B C$.


Fig. 177.

Bisect AC in D (Prob. i). With D as centre, and radius DA, draw the semicircle AEC. At B draw BE perpendicular to AC (Prob. 5). Join DE. From B draw BF perpendicular to DE . AD is the Arithmetic, BE the Geometric, and EF the Harmonic mean between the two lines as required.


Fig. 178.
123. Taking the given line $A B$ as the unit ; find lines representing $\sqrt{2}$ and $\sqrt{3}$.
Draw AC perpendicular to $A B$, and of the same length (Prob. 5). Join $C B$. $C B=\sqrt{2}$.

Draw CD perpendicular to $C B$, and equal in length to $A B$ and $A C$. Join $D B . \quad D B=\sqrt{3}$.

If $A B$ is the edge of a cube, CB is the diagonal of its face, and $D B$ the diagonal of the cube, which are therefore to one another as $I: \sqrt{2}: \sqrt{3}$.

Table of Foreign Road Measures and their Equivalents in English Yards.

|  |  |  | English Yards. |
| :---: | :---: | :---: | :---: |
| Austria, | - - | mile | 8297 |
| Bavaria, - | - - |  | 8059 |
| Belgium, - | - - | kilomètre | 1094 |
| Berne, | - - | league | 5770 |
| China, - | - - | li | 609 |
| Denmark, | - - | mile | 8238 |
| England, - | - - |  | 1760 |
| France, - | - - | kilomètre | 1094 |
| Germany, | - - | mile | 8 IOI |
| Greece, - | - - | " | 1640 |
| Holland, - | - - | " | 1094 |
| India (Bengal), | - - | coss | 2000 |
| Italy, - | - - | mile | 2025 |
| Netherlands, | - - | kilomètre | 1094 |
| Norway, - | - - | mile | 12,182 |
| Persia, - | - - | parasang | 6076 |
| Portugal, - | - - | mile | 2250 |
| Prussia, | - - | " | 8238 |
| Russia, | - - | verst | 1167 |
| Siam, | - - | röeneng | 4204 |
| Spain, | - - | mile | 1522 |
| Sweden, | - - |  | 11,690 |
| Turkey, - | - - | berri | 1827 |

## EXERCISES.

1. Draw three lines $1.25^{\prime \prime}, 2.3^{\prime \prime}$, and $2.75^{\prime \prime}$ respectively, and find their greater fourth proportional.
2. Draw two lines $2.7^{\prime \prime}$ and $1.5^{\prime \prime}$ in length, and find their less third proportional.
3. Draw a line $2.5^{\prime \prime}$ in length, and produce it so that its extra length shall be in proportion to its original length as $3: 5$.
4. Draw two lines $2 \frac{1^{\prime \prime}}{4}$ and $\frac{7^{\prime \prime}}{8}$ in length, and find their mean proportional.
5. Construct a triangle on a base $3.25^{\prime \prime}$ in length, so that its three angles are in the proportion of 3,4 , and 5 .
6. Divide a line $3 \cdot 4^{\prime \prime}$ in length in extreme and mean proportion.
7. Divide a line $2.75^{\prime \prime}$ in length so that one part is in proportion to the other as $2: 4$.
8. Draw two lines $1.25^{\prime \prime}$ and $2.3^{\prime \prime}$ respectively, and find their greater third proportional.
9. Draw three lines $3^{\prime \prime}, 24^{\prime \prime}$, and $1 \frac{7^{\prime \prime}}{8}$ in length, and find their less fourth proportional.
10. The base of a triangle is $1.65^{\prime \prime}$ long, and the angles at the base are $88^{\circ}$ and $53^{\circ}$. Construct the triangle, and find a fourth proportional less to the three sides. Measure and write down the length of this line. (The angles should be found from the protractor or scale of chords.)
(April, 'or.)

## CHAPTER VIII.

## PLAIN SCALES, COMPARATIVE SCALES, AND DIAGONAL SCALES.

On a drawing representing a piece of machinery the scale is written thus : Scale $\frac{1}{4}$ full size. From this we know that every inch on the drawing represents 4 inches on the actual machine, so the relation between any part represented on the drawing and a corresponding part in the real object is as $1: 4$ or $\frac{1}{4}$. This is called the representative fraction.

A drawing representing a building has drawn upon it a scale; e.g.-Scale $\frac{1}{4}$ of an inch to a foot. One-quarter of an inch is contained forty-eight times in I foot, so the R.F. is $\frac{1}{48}$.

On a large drawing showing a district the scale is written thus : R.F. ${ }_{1 \frac{1}{760}}$. As there are 1760 yards to a mile, it is evident that every 3 feet on the drawing is equal to 1 mile on the land represented. This, of course, is a very large scale.

Our Ordnance Survey Office publishes a map of 25 inches to a mile, which is useful for small districts or estates ; one of 6 inches to a mile, useful for maps of parishes ; and one of 1 inch to a mile, useful for general purposes.

The R.F. for the last would be $\frac{1}{63.36}$.

$$
\begin{aligned}
& \text { mile. yards. feet. inches. } \\
& 1=1760=5280=63,360 \text {. }
\end{aligned}
$$

124. To construct a scale 4 inches long, showing inches and tenths of an inch. Fig. 179.


Fig. 179.

Draw a line 4 inches long, and divide it into four equal parts, each of which will be I inch. At the end of the first inch mark the zero point, and from this point mark the inches to the right $\mathrm{I}, 2,3$. These are called primary divisions, and the amount by which they increase is called the value of the scale length.

The division left of the zero point has to be divided into ten equal parts. The best way to do this is to take a piece of paper and set off along its edge ten equal divisions of any convenient size (Fig. 179). Produce the perpendicular marking the division at the zero point, and arrange this piece of paper so as to fit in exactly between the end of the division and this perpendicular line. If we now draw lines parallel to the perpendicular at the zero point, they will divide the inch into ten equal parts. These are called the secondary divisions ; they have the same zero point as the primary, and their numbers increase from this poini by the value of their scale length.

This scale will measure inches to one decimal place. Supposing we wish to measure 3.6 inches, that is 3 primary and 6 secondary divisions. Place one point of the dividers on division 3 of the primary parts, and open them till the other point reaches the division 6 of the secondary divisions.
125. To construct a scale of $\frac{1}{36}$, or 1 inch to equal 3 feet.


Fig. 181.

128. To construct a comparative scale of English miles. Scale to measure 80 miles. Fig. 183.
A French league $=4262.84$ English yards.

$$
\begin{gathered}
\begin{array}{c}
\text { French } \\
\text { leagues. }
\end{array} \begin{array}{c}
\text { English } \\
\text { miles. }
\end{array} \\
30=\frac{4262.84}{1760} \times 30 \\
\frac{4262.84}{1760} \times 30 \\
: 80:: 4: x
\end{gathered}
$$

whence
$x=\frac{4 \times 80 \times 1760}{4262.84 \times 30}=4.4$ inches nearly.
Draw a line of this length, and place the zero point at the left-hand end and 80 at the other extremity. Divide this line into eight equal divisions : each of these primary divisions will represent io miles.

For the secondary divisions, set off one of the primary divisions to the left of the zero point, and divide it into ten equal divisions: each of these will represent I mile. The representative fraction of both the French and English scales will of course be the same.

On a Russian map a scale of versts is shown, as Fig. 184, by measuring which by an English scale 120 versts $=4$ inches.
129. To construct a comparative scale of

English miles. Scale to measure 80 miles. Fig. 184.
A Russian verst $=1167$ English yards.

$$
120 \text { versts }=\frac{1167 \times 120}{1760} \text { miles }
$$

$$
\therefore \frac{1167 \times 120}{1760}: 80:: 4: x
$$




Fig. 184.
whence

$$
x=\frac{4 \times 80 \times 1760}{1167 \times 120}=4 \text { inches nearly } .
$$

Draw a line of this length, and divide it into eight equal divisions : each of these primary divisions will represent 10 miles. Place the zero point at the left-hand end of the line, and figure the divisions towards the right $10,20,30$, etc. Set off one of the primary divisions to the left of the zero point, and divide it into ten equal divisions : each of these will represent I mile.

## Diagonal Scales.

In the preceding scales we have only primary and secondary divisions, and if we wish to measure a fractional proportion of a secondary division, we cannot do it with any accuracy ; but by means of a diagonal scale we are enabled to measure hundredths of primary divisions, as will be seen from the following scale.
130. To construct a diagonal scale 3 inches long, to measure inches, tenths of inches, and hundredths of inches. Fig. 185a.
Draw a rectangle ABDC 6 inches long and about $\frac{1}{4}$ inches wide, and divide it into six equal parts. At the end of the first


Fig. ${ }^{8}{ }_{5}$ a.


Fig. 185b.
N.B.-These two figures are half the size described in the text, and should be drawn full size by the student.
division from A fix the zero point, and to the right of this figure each division $\mathrm{I}, 2,3,4$, and 5. Divide AC into ten equal parts, and figure them from A towards $C$, then draw lines parallel to $A B$ from one end of the scale to the other. Divide

Ao into ten equal divisions, and figure them from o towards A. Join 9 to C , and from each of the other divisions between A and o draw lines parallel to 9C. Note.-The divisions between $o$ and $B$ are primary, between $o$ and A secondary and between A and C tertiary.

To take off from this scale a measurement equal to 2.73 inches, we place one point of the dividers on the primary division figured 2 , and the other on the secondary division figured 7, but both points must be on the line that is figured 3 on AC. The points are marked by a small circle on the scale.

To take off 3.17 inches, place one point of the dividers on the primary division 3 , and the other on the intersection between the secondary division 1 and the line 7 on AC. These points are shown by two crosses on the scale.
131. To construct a diagonal scale showing miles, furlongs, and chains, to show 6 miles, R.F. $=\frac{\overline{6} 3}{} \frac{1}{60}$. Fig. 185 b .

$$
\text { I mile }=8 \text { furlongs. }
$$

I furlong $=10$ chains.
The length of scale $=\frac{1}{63 \frac{1}{60}}$ of 6 miles $=6$ inches.
In this scale there will be six primary, eight secondary, and ten tertiary divisions.

Construct a rectangle ABDC 6 inches long and about $I_{4}^{\frac{1}{4}}$ inches wide, and divide it into six primary divisions. Place the zero point $o$ at the end of the first division from $A$, and divide Ao into eight secondary divisions, figured from o towards $A$. Divide AC into ten equal divisions, and figure them from $A$ to C. Join the secondary division figured 7 to the point $C$, and from each of the other secondary divisions draw lines parallel to 7 C , thus completing the scale.

To take off from this scale 2 miles, 5 furlongs, and 7 chains, take one point on the primary division 2 , and the other where the line from the secondary division 5 intersects the division 7 on AC. These two points are marked by two small circles on the scale.

To take off 3 miles, 2 furlongs, and 3 chains, one point will be on the primary division 3 , and the other where the secondary division 2 intersects the tertiary division 3. These points are marked by two small crosses on the scale.
132. To take off any number to three places of figures from a diagonal scale.
On the parallel indicated by the third figure, measure from the diagonal indicated by the second figure to the vertical line indicated by the first.


Comparative Diagonal Scales are very useful for transferring the value of quantities in one measure to another. To make the scales less cumbersome they are so arranged that in some cases the number to be taken off must be halved. With this proviso, any quantity may be converted from one scale to another. The number expressing the quantity in one unit is taken off on the scale for that unit, and the number expressing it in the other unit is at once read off on the parallel scale.

For example, a length of 638 miles. Its half, 319 miles, corresponds to 513 kilometres, so that 638 miles corresponds to 1026 kilometres.

The diagonal scale generally found in instru-ment-boxes is shown in Fig. 187.


Fig. 187.

It consists of two diagonal scales. In one, the distance between the primary divisions is half an inch, and in the other a quarter of an inch.

There is a small margin on each side of the scale for figures : on one side the half inches are figured, and the quarter inches on the other.

One primary division at each end is divided into ten secondary divisions, and there are ten tertiary divisions drawn from one end of the scale to the other.

The primary divisions being taken for units, to set off the numbers 5.36 by the diagonal scale. This measurement is shown by two crosses on the scale.

If we reckon the primary divisions to stand for tens, the dimension would have one place of decimals, e.g. to take off 36.4 from the diagonal scale. These points are shown on the scale by two small circles.

The primary divisions being hundreds, to take off 227. This dimension is shown on the scale by two small squares.

## Proportional Scales.

These are used for enlarging or reducing a drawing in a given proportion: three varieties are here illustrated.

The simplest form is that shown in Fig. 188. Suppose we wish to enlarge a drawing in the proportion of $3: \mathrm{I}$.

Draw the line AB of convenient size, to suit the measurements on the drawing, and produce it to C ; make $B C$ one-third of $A B$. On $A B$ erect the perpendicular BD any length, and join AD and DC . Divide BD into any number of equal parts, and draw lines parallel to AB. These lines are simply a guide to enable the measurements to be made parallel to the basee.g. on placing a measurement from


Fig. 188. the original drawing on the scale we find it occupies the position of ef: the distance between $e$ and $g$ will then give the length of the measurement to the enlarged scale, i.e. in the proportion of $3: 1$. We proceed in the same manner with every measurement we wish to enlarge.

Should we wish to reduce a drawing in the same proportion, viz. I : 3, the original measurements would be placed on the left-hand side of the scale, and the required proportion taken from the right-hand side.

In Fig. 189 we have a series of measurements- $\mathrm{A} i, \mathrm{~A} h, \mathrm{~A} g$, etc.-which we wish to enlarge, say in the proportion of $3: 2$.


Fig. ${ }^{8} 8$.

Draw the line $A B$ any convenient length to suit size of drawing. From B draw BD perpendicular to $A B$. Produce $A B$ to $C$, and make BC equal to half of AB . With centre A , and radius AC , draw an arc till it meets BD in D ; and join DA. From each of the points, $i, h, g, f$, etc., draw lines parallel to BD. The distances $A i^{\prime}$, $\mathrm{A} h^{\prime}, \mathrm{A} g^{\prime}$, etc., will then give the original measurements to the enlarged scale of $3: 2$.

To reduce the original drawing in the same proportion, i.e. 2:3. With $A$ as centre, and radius AB , draw the arc BE . From E draw the line EF parallel to $B D$. $A F$ will then represent $A B$ reduced in the proportion of $2: 3$, and so on with any other measurement that we may require.

## EXERCISES.

1. Construct a scale to measure feet and inches; the R.F. to be $\frac{1}{36}$, and its scale length value 15 feet.
2. Construct a scale to measure yards and feet, the R.F. to be $\frac{1}{72}$, to measure 18 yards.
3. Construct a diagonal scale to measure feet and inches, R.F. $\frac{1}{72}$, to measure 36 feet. Take off a length of $17^{\prime} 9^{\prime \prime}$.
4. On a map, a distance known to be 20 miles measures $10^{\prime \prime}$; construct a diagonal scale to measure miles and furlongs, long enough to measure 12 miles.
5. Construct a diagonal scale to measure yards and feet, R.F. $\frac{1}{1 \frac{1}{8}}$, to measure 30 yards.
6. On a map showing a scale of kilomètres, 60 are found to equal $3^{\prime \prime}$ : What is the R.F.? Construct a comparative scale of English miles, to measure 100 miles.
7. A line $4 \frac{3^{\prime \prime}}{4}$ long represents a distance of $4^{\prime}$. Construct a scale by which feet and inches may be measured up to 4 feet. The scale must be neatly finished and correctly figured.
(April, '96.)
8. Construct a scale to show yards and feet, on which $3 \frac{12^{\prime \prime}}{}$ represent 8 yards. Make the scale long enough to measure 10 yards, and finish and figure it properly.
(May, '97.)
9. Construct a scale of $7 \frac{\frac{1}{2}^{\prime}}{}$ to $\mathrm{I}^{\prime \prime}$, by which single feet may be measured up to $30^{\prime}$. The scale must be neatly finished and correctly figured.
(June, '97.)
10. The diagram represents an incomplete scale of feet (Fig. 190.) Complete the scale so that distances of $\mathbf{z}^{\prime}$ may be measured by it up to $50^{\prime}$. The scale must be properly finished and figured.
(April, '98.)


Fig. 190.
11. Construct a scale one-tenth of full size, to measure feet and inches up to 5 feet. The scale must be properly finished and figured.
(June, '98.)
12. Construct a scale of feet and inches one-ninth ( $\left(\frac{1}{9}\right)$ of full size long enough to measure 4 ft . The scale must be properly finished and figured, and should not be "fully divided" throughout i.e. only one distance representing Ift. should be divided to show inches. (April,' '99.)
13. The line AB ( 2.75 inches long) represents a distance of $2 \frac{1}{2} \mathrm{ft}$. Make a scale by which feet and tenths of a foot may be measured up to 4 ft . The scale must be properly finished and figured, and should not be "fully divided" throughout, i.e. only one distance representing I foot should be divided to show tenths.
(June, '99.)
14. The given line AB ( 3.2 inches long) represents a distance of 35 ft . Construct a scale by which single feet may be measured up to 40 ft . The scale is not to be "fully divided" i.e. single feet are not to be shown throughout the whole length, and it must be properly finished and figured.
(April, 'oo.)
15. A drawing is made to a scale of $\mathrm{I}^{\frac{1^{\prime \prime}}{}}$ to $\mathrm{I}^{\prime}$, and another drawing is required on which the dimensions shall be three-quarters of those on the fir:t drawing. Make a scale for the second drawing to show feet and inches up to 5 ". The scale is not to be "fully divided" (i.e. only one length of 1 ' is to be divided to show inches) and it must be properly finished and figured.
(June, 'oo.)
16. Construct a scale $\frac{1}{3}$ of full size, by which feet and inches may be measured up to 2 feet. Show also distances of $\frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ by the diagonal method. Finish and figure the scale properly, and show by two small marks on it how you would take off a distance of $\mathrm{I}^{\prime} 7 \frac{\frac{1}{2}^{\prime \prime}}{}$. (April, 'oI.)
17. Draw a diagonal scale, one-tenth of full size, by which centimetres may be measured up to one metre. Mark on the scale a length of 47 centimetres.

One metre ( 100 centimetres) may be taken as $39^{\prime \prime}$.
(May, 'O3.)

## CHAPTER IX.

## InSTRUMENTS FOR MEASURING ANGLES, ETC.

A protractor is an instrument used for measuring or setting off angles; it may be either semicircular or rectangular in shape, as

shown in Fig. 191. The point $C$ marks the centre from which the radiating lines are drawn, and corresponds with the centre of the circle.

The degrees are numbered in primary divisions, equal to ten degrees each, on the outside line from $A$; and on the inside line from B. In the actual instrument each of these primary divisions is subdivided into ten secondary divisions, each of which represents one degree. Only one of these is divided in the figure.
133. To construct a scale of chords.

A scale of chords is constructed in the following manner (Fig. 192). Draw the lines AC and CD perpendicular to each
other. With C as centre, draw any quadrant AED , and divide the arc into degrees (only the primary divisions are shown in the figure). Join AD. With A as centre, and each of the primary divisions as radii, draw arcs cutting the chord AD, which will form the scale of chords.

To use this scale in setting off an angle-for example, to draw a line that will make an angle of $40^{\circ}$ with line $C B$ (Fig. 192).

With C as centre, and


Fig. 192. radius equal to A 60 on the scale of chords, draw an arc BFD. With a pair of dividers, take the distance A40 from the scale, and set it off on the arc BF from B. Join FC. Then FCB will be the angle of $40^{\circ}$ required.

Note.-A60 is always equal to the radius of the quadrant from which the scale of chords is constructed.
134. To construct any angle without a protractor.

Draw CD perpendicular to $A B$. With $C$ as centre, and CA as radius, draw the semicircle ADB . Trisect the angle DCB in E and F (Prob. 13). Trisect the angle ECB in H and K (Prob. 14). Bisect FK in L (Prob. 12). Then $\mathrm{DE}=30^{\circ}, \mathrm{EH}=20^{\circ}, \mathrm{HF}=10^{\circ}$, and $\mathrm{FL}=5^{\circ}$. Therefore between $D$ and $B$ we can construct any angle that is a multiple of $5^{\circ}$.

Divide the angle ACD into five equal parts by the radii


Fig. 193. from M, N, O, and P (Prob. 12I). From A set off AR equal to DE. As $\mathrm{AN}=36^{\circ}$ and $\mathrm{AR}=30^{\circ}, \mathrm{AN}-\mathrm{AR}=6^{\circ}, \mathrm{MR}=12^{\circ}$, $\mathrm{AM}=18^{\circ}$, and $\mathrm{RO}=24^{\circ}$. Therefore between A and D we can construct any angle that is a multiple of $6^{\circ}$.

If we subtract the multiples of $5^{\circ}$ from those of $6^{\circ}$ we can obtain any desired angle, e.g.-

$$
\begin{array}{r}
6-5=1^{\circ} \\
12-10=2^{\circ} \\
18-15=3^{\circ} \\
24-20=4^{\circ} \\
30-25=5^{\circ} \\
\text { etc. etc. etc. }
\end{array}
$$

All the regular polygons, with the exception of two-the heptagon and undecagon-can be constructed with angles that are multiples of $5^{\circ}$ or $6^{\circ}$.

If the polygon is to be inscribed in a circle, the angle would be set off at the centre of the circle ; but if one side of the polygon is given, the angle would be set off externally, as shown in Fig. 74, page 31.

The exterior angle of a Pentagon is $72^{\circ}$ a multiple of $6^{\circ}$.

| $"$ | $"$ | Hexagon | $" 60^{\circ}$ | $"$ | $6^{\circ}$. |
| :--- | :--- | :---: | :---: | :--- | :--- |
| $"$ | $"$ | an Octagon | $" 45^{\circ}$ | $"$ | $5^{\circ}$. |
| $"$ | $"$ | a Nonagon | $" 40^{\circ}$ | $"$ | $5^{\circ}$. |
| $"$ | $"$ | Decagon | $" 36^{\circ}$ | $"$ | $6^{\circ}$. |
| $"$ | $"$ | Duodecagon | $" 30^{\circ}$ | $" 5^{\circ}$ and $6^{\circ}$. |  |

## The Sector.

The Sector is an instrument of great utility for facilitating the work of Practical Geometry. It consists of graduations on the two radii of a foot-rule, and it is used by measuring the arc between the graduations. Hence its name. The legs can be opened to contain any angle up to a straight line.

In the illustration (Fig. 194) only the lines most used in Practical Geometry are shown : viz. line of lines, marked L on each leg; a pair of lines of chords, marked C ; and a line of polygons, marked POL, on the inner side of each leg.

The sectoral lines proceed in pairs from the centre of the hinge along each leg, and although the scales consist of two or three lines, parallel to the sectoral lines, all measurements must be made on the inner lines of each scale, i.e. the lines that radiate from the centre.

When the measurement is confined to a line on one leg of the sector, it is called a lateral distance; but when it is taken from a


Fig. 194.
point on a line on one leg to a similar point on a corresponding: line on the opposite leg, it is called a transverse distance.

Simple proportion.-Let AB and AC (Fig. 195) represent a pair of sectoral lines, and $B C$ and $D E$ two transverse measurements taken between this pair of lines; then $A B$ is equal to $A C$, and AD to AE , so that $\mathrm{AB}: \mathrm{AC}:: \mathrm{AD}: \mathrm{AE}$, and the lines $A B: B C:: A D: D E$.

## The Line of Lines.

The primary divisions only are shown in the illustration ; in the real instrument, each


Fig. 195. of these is subdivided into ten secondary divisions
135. To find the fourth proportional to three given lines.

From the centre, measure along one leg a lateral distance equal to the first term ; then open the sector till the transverse distance
between this point and a corresponding point on the other leg is equal to the second term ; then measure from the centre along one leg a lateral distance equal to the third term; the transverse distance from this point to a corresponding point on the opposite leg will then give the fourth term.

Example.-To find the fourth proportional to the numbers 3 , 4 and 9. From the division marked 3, which is the first term, open the sector till the distance between this point and the corresponding division on the other leg is equal to 4 divisions: this will be the second term. Then 9 being the third term, the transverse distance between the corresponding divisions at this point will give the fourth term, viz. 12.
136. To find the third proportional to two given lines or numbers.

Make a third term equal to the second, then the fourth term will give the required result.

## 137. To bisect a given line.

Open the sector till the transverse distance between the end divisions, 10 and ro, is equal to the given line ; then the transverse distance between 5 and 5 will bisect the given line.
138. To divide a given line $A B$ into any number of equal parts.

For example, eight (Fig. 196). When the number of parts is a power of 2 , the division


Fig. 196. is best performed by successive bisections. Thus, make $A B$ a transverse distance between 10 and 10 , then the distance between 5 and 5 will give $A C=$ half $A B$. Then make the transverse distance between 10 and $10=A C$, the distance between 5 and 5 will then give $\mathrm{AD}=$ one quarter of AB . By repeating the operation each quarter will be bisected, and the given line divided into eight equal parts as required.

When the number of divisions are unequal, -for example, seven (Fig. 197), -make the transverse distance between 7 and


Fig. 197.
7 equal to the given line $A B$; then take the distance between 6 and 6 , which will give AC. The distance $C B$ will then divide the line into seven equal parts.

## 139. How to use the sector as a scale.

Example.-A scale of 1 inch equals 5 chains. Take one inch on the dividers, and open the sector till this forms a transverse distance between 5 and 5 on each line of lines ; then the corresponging distances between the other divisions and subdivisions will represent the number of chains and links indicated by these divisions: for instance, the distance between 4 and 4 represents 4 chains, $6.5=6$ chains 50 links, $3.7=3$ chains 70 links, etc.

$$
\text { Note.-I chain is equal to } 100 \text { links. }
$$

140. To construct a scale of feet and inches, in which $2 \frac{1}{2}$ inches shall represent 20 inches.

Make the transverse distance between 10 and io equal to $2 \frac{1}{2}$ inches ; then the distance between 6 and 6 will represent 12 inches. Make AB (Fig. 198) equal to this length. Bisect this distance


Fig. 198.
in C, as described for Fig. 196 ; then bisect AC and CB in D and $E$ in the same manner. Take the transverse distance between 5 and 5, which will give AF 10 inches; EF will then trisect each of the four divisions already obtained. AB will then be divided into twelve divisions, which will represent inches. Produce the line AB to H , and make BH equal to AB . BH will then represent one foot.
141. How the sector may be used for enlarging or reducing a drawing.
Let ABC (Fig. 199) represent three points in a drawing, let it be required to reduce this in the proportion of 4 to 7 . Make the transverse distance between 7 and 7 equal to $A B$; then take the distance between 4 and 4 , and make DE equal to this length. Also niake the distance between 7 and 7 equal to


Fig. 199.
AC ; then take the distance between 4 and 4 , and from D as centre, with this distance as radius, describe an arc. In the same manner make the distance between 7 and 7 equal to BC ; then with a radius equal to 4,4 , describe another arc from E , cutting the other arc in F. Join EF and DF. Then DEF will be a reduced copy of $A B C$, in the proportion of $4: 7$ as required.
142. To enlarge a drawing in the proportion of 7 to 4.

In this instance the sector would be opened so that the transverse distance between 4 and 4 should represent the original measurements, while those required for the copy would be taken between 7 and 7 .

## The Line of Chords.

In the scale of chords already described (Prob. 133) we are limited to one radius in setting off angles-viz. a radius equal to the 60 marked on the scale ; in the double line of chords on the
sector there is no such limitation-we can set off any radius equal to the transverse distance between the two points 60 and 60 , from their nearest approach to each other up to the fullest extent the opening of the sector will admit of.

## 143. To construct an angle of $50^{\circ}$.

Open the sector at any convenient distance. Take the transverse distance between the points 60 and 60 , and construct an arc with this radius. Let AB (Fig. 200) represent this radius. Now take the transverse distance between 50 and 50 , and set it off from $B$ on the arc, which will give the point C. Join AC. Then BAC will be $50^{\circ}$, as required.

A greater angle than $60^{\circ}$ cannot


Fig. 200. be taken from the sector with one measurement ; if the angle to be measured is more than $60^{\circ}$, successive measurements must be taken.
144. On an arc $\frac{3}{4}$ inch in radius, to construct an angle of $125^{\circ}$.

Make the transverse distance between the points 60 and 60 $\frac{3}{4}$ inch. Let AB (Fig. 201) represent this distance. Describe an arc with AB as radius. Take the distance between the points 50 and 50 from the sector, and set it off on the arc from B to C. Also take the distance from 40 to 40 , and set it off from $C$ to D. Then take the distance between 35 and 35 , and set it


Fig. 20 . off from D to E. Join EA. Then the angle BAE will be $125^{\circ}$. $50^{\circ}+40^{\circ}+35^{\circ}=125^{\circ}$.

## 145. To construct an angle of $3^{\circ}$ on the same arc.

With the sector open at the same angle as before, take the transverse distance between the points 47 and 47 , and set it off on the arc from B to H . Join HA and CA. Then HAC will be $3^{\circ}$ as required. $50^{\circ}-47^{\circ}=3^{\circ}$

## The Line of Polygons.

This pair of lines is used for dividing a circle into any number of equal parts between four and twelve, by joining which the regular polygons are formed. The transverse distance between the points 6 and 6 is always used for the radius of the circle to be divided; because the radius of a circle containing a six-sided figure, i.e. a hexagon, is always equal to one side of the figure.

Open the sector till the transverse distance between 6 and 6 is equal to the radius of the circle ; then the distance between the points 4 and 4 will divide the circle into four equal parts, the distance between 5 and 5 into five equal parts, and so on up to twelve.

If it be required to construct a polygon on a given straight line, open the sector till the transverse distance between the numbers answering to the number of sides of the required polygon shall equal the extent of the given line, then the distance between the points 6 and 6 will give the radius of the circle to be divided by the given line into the required number of equal parts.
146. On a given line 1 inch in length, to construct a heptagon.

Open the sector till the transverse distance between the points 7 and 7 shall equal I inch; the distance between the points 6 and 6 will then give the radius of a circle, to which the given line will form seven equal chords.

## EXERCISES.

1. On a line $4^{\prime \prime}$ long, draw a semicircle, and upon it set out the primary divisions of a protractor, by construction alone.
2. Construct a scale of chords from the protractor set out in the preceding question.
3. Make a scale of chords of $2^{\prime \prime}$ radius, to read to $10^{\circ}$ up to $90^{\circ}$. The scale must be finished and figured. At the ends of a line $2 \frac{1}{2}{ }^{\prime \prime}$ long construct, from the scale, angles of $20^{\circ}$ and $70^{\circ}$ respectively. (April, '99.)

## CHAPTER X.

THE CONSTRUCTION OF SIMILAR FIGURES. PRINCIPLES OF SIMILITUDE.

## Similar Figures.

Similar figures have their angles equal and their corresponding sides proportional.
All regular figures-such as equilateral triangles, squares, and regular polygons-are similar. Other quadrilateral figurestriangles and irregular polygons-can be constructed similar to given ones by making their angles equal.
147. To construct within a given triangle $A B C$, and equidistant from the sides of it , a similar triangle, the base of which is equal to the given line D . Fig. 202.
Bisect the angles BAC and ACB by lines meeting at the centre E (Prob. 12). Join EB. On the line $A B$ set off $A F$ equal to the given line D. From F draw a line parallel to AE till it cuts EB at G (Prob. 3). From G draw a line parallel to BC till it cuts EC at H. From H


L draw a line parallel to AC till it cuts EA at K. Join KG. KGH will be the similar triangle required.
148. To construct about a given triangle $A B C$, and equidistant from its sides, a similar triangle, the base of which is equal to a given line L. Fig. 202.
Set off on the base $A B$ produced, AN equal to the given line L. From N draw a line parallel to EA (Prob. 3) till it meets EB produced at O. From O draw a line parallel to AB till it meets EA produced at M. From M draw a line parallel to AC till it meets EC produced at P. Join PO. Then MOP will be the similar triangle required.
149. To construct within a given square ABDC , and equidistant
 from its sides, a square, one side of which is equal to the given line E .
Draw the diagonals AD and CB. From $A$ set off $A F$ along AB equal to the given line E . From $F$, parallel to AD , draw a line till it meets CB at G (Prob. 3). With $M$ as centre, and radius MG , set off the points H , K , and L , and join GH, HK, B KL, and LG. Then HGLK will be the square required.
Fig. ${ }^{203}$.
150. To construct a triangle similar to a given triangle CDE , and having its perimeter equal to a given straight line $A B$.


Fig. 204.

On the given line AB construct a triangle ABF similar to the given triangle CDE, by making the angles at A and B equal to the angles at C and D respectively. Bisect the angles at A and $B$ by lines meeting at $G$. From G draw a line parallel to FB till it meets $A B$ at $L$; and also a line parallel to AF till it meets AB at H . Then HLG will be the triangle required.

## Principles of Similitude.

Draw a rectangle ABDC , and join each angle to any point E. Bisect EB in G, EA in F, ED in K, and EC in H. Join FG, GK, KH, and HF, then FGKH will be a rectangle with sides one-half the length of the rectangle ABCD . If we draw the diagonals BC and GH , we shall find they are parallel to each other, and that


Fig. 205. $B C: G H$ as $2: 1$. If we take any point $L \cdot i n B D$ and join it to point E, LE will intersect GK in M, and will divide GK in the same proportion as BD is divided.

On the principle of this problem, we can draw a figure similar to a given figure, and having any proportion desired, e.g.-

If we wish to draw a rectangle having sides equal to onethird of a given rectangle, we should trisect the lines drawn from the angles of the given rectangle to E . The point E is called the centre of similitude of the two figures ABDC and FGKH, which in this instance are said to be in direct similitude. The following problem shows the principle of inverse similitude.
151. To draw a trapezium similar to a given trapezium ABDC , with sides two-thirds the length of those of the given trapezium, in INVERSE SIMILITUDE.
Take any point E in a convenient position. Produce a line from A through E to K , making EK equal to twothirds of AE (Prob. 117). Proceed in the same manner with each of the points $\mathrm{B}, \mathrm{C}$, and


Fig. 206.

D , which will give the points $\mathrm{H}, \mathrm{G}$, and F . Join KH, HF, FG, and GK. Then FGHK will be the trapezium required.
152. To draw an irregular polygon similar to a given polygon, but with sides two-thirds the length of those of the given polygon ABCDEFG.
It is not necessary that the centre of similitude should be
 taken outside the figure ; if more convenient, we can use one of the angles of the figure, e.g. let A be the centre of similitude. Draw lines from all the angles of the polygon to A. Make AK two-thirds of AD (Prob. II7), and divide all the other lines in the same proportion, which will give the points $\mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}, \mathrm{H}$, and K. Join NM, ML, LK, KH, and HO , which will give the polygon required.

Note.-The figures are similar, and the corresponding sides are parallel to each other.
153. To draw an irregular pentagon similar to a given pentagon ABCDE, but with sides one-half the length of those of
 the given pentagon, without using any centre of similitude.
Draw FG in any convenient position parallel to AB , and half its length (Prob. 4). From G draw GH parallel to BC and half its length. Proceed in the same way with the remaining sides, which will give the pentagon required.
154. To draw a curve or pattern similar to a given figure, but to two-thirds the scale.
Enclose the given figure in a convenient rectangular figure ABDC , and divide the sides of the rectangle into equal parts
(Prob. 9). Join these divisions, which will divide the rectangle into a number of equal squares or rectangles. Draw another


Fig. 209.


Fig. 210.
rectangle EFGH with sides two-thirds the length of the rectangle enclosing the given figure, and divide it in a similar manner.

Draw the curves to intersect these smaller divisions in the same places as the larger divisions are intersected by the given figure.

Note.-This method is used for enlarging or reducing maps or drawings to any scale.

## EXERCISES.

1. Draw a regular hexagon on a side of $\mathrm{I}^{\prime \prime}$; construct a similar figure on a side of $\frac{1^{\prime \prime}}{}{ }^{\prime \prime}$, using one angle as the centre of similitude.
2. Draw a rectangle with sides of $27^{\prime \prime}$ and $1.5^{\prime \prime}$; construct a similar figure by inverse similitude, with sides in the proportion to those given as $3: 5$.
3. Draw a regular pentagon in a circle of $23^{\prime \prime}$ diameter ; construct a similar figure by direct similitude, with sides in the proportion to those of first pentagon as $4: 7$.
4. Make an irregular pentagon ABCDE from the following data : Sides: $\mathrm{AB}=2 \frac{1^{\prime \prime}}{8}, \mathrm{BC}=1 \frac{1_{4}^{\prime \prime}}{4}, \mathrm{CD}=2 \frac{3{ }^{\prime \prime}}{\frac{1}{2}}, \mathrm{DE}=1^{\prime \prime}$. Angles: $\mathrm{ABC}=105^{\circ}, \mathrm{ABE}=30^{\circ}, \mathrm{BAE}=105^{\circ}$.
Then make a similar figure in which the side corresponding to AB is I ${ }_{\frac{3}{3}}{ }^{\prime \prime}$ long.
(June, '98.)
 $2^{\prime \prime}$ long. Make a similar rectangle having its shorter sides $I \frac{1^{\prime \prime}}{}$ long.
(April, '96.)
5. Construct a triangle, sides $\mathrm{I}^{\frac{1}{2}}{ }^{\prime \prime}, \mathrm{I}^{\frac{7^{\prime \prime}}{8}}$, and $2 \frac{11^{\prime \prime}}{}$ long, and a similar triangle having its longest side $2 \frac{1}{2}{ }^{\prime \prime}$ long. Measure and write down the number of degrees in each of the angles. (June, '97.)
6. Construct a figure (Fig. 21I) similar to that given in the diagram, but having the side corresponding to $\mathrm{AB} \mathbf{2}^{\prime \prime}$ long.
(May, '97.)


Fig. 2ir.


Fig. 212.


Fig. 213.
8. Construct a figure similar to the given figure (Fig. 212), but having the distance corresponding to CD $\mathrm{r}^{\prime \prime}$ long.
9. Make a figure similar to the given one (Fig. 213), but having the length corresponding to $\mathrm{CD} 2^{\prime \prime}$ long.

## CHAPTER XI.

## CONIC SECTIONS.

A conic section is obtained by intersecting a cone by a plane. There are five different sections to a cone, viz.:
r. A triangle, when the plane cuts the cone through its axis.
2. A circle, when the plane cuts the cone parallel to its base, as at A, Fig. 214.
3. An ellipse, when the plane cuts the cone obliquely, without intersecting the base, as at B.
4. A parabola, when the plane cuts the cone


Fig. 214. parallel to one side, as at C.
5. An hyperbola, when the cone is cut by a plane that is perpendicular to its base, i.e. parallel to its axis, as at D , or inclined to the axis at a less angle than the side of the cone.

These curves can be drawn with the greatest accuracy and facility by the following arrangement. Cut a circular opening in a piece of thin card-board or stiff paper, and place it a short distance from a lighted candle ; this will form a cone of light (Fig. 215). If we place a plane, e.g. a piece of paper pinned to a drawing-board, so as to allow the light coming through the circular aperture to fall upon it, we can, by placing it in the


Fig. 215. several positions, intersect this cone of light so as to form the required sections, which can then be traced. C is the candle, A is the circular aperture, and P the plane.

In Fig. 215 the plane is parallel to the aperture, so the section obtained is a circle.


Fig. 216.
Fig. 217.
If the plane is placed obliquely to the aperture, as in Fig. 216, the section obtained is an ellipse.

By placing the plane parallel to the side of the cone, as in Fig. 217, we get as section, the parabola.

If we place the plane at right angles to the aperture, we obtain the hyperbola, Fig. 218.

By adjusting the positions of the candle, aperture, and plane, we can obtain a conic section to suit any required condition, both as to shape or size.

A truncated cone or frustum is the part of the cone below any section as A or B, Fig. 214.

## The Ellipse.

An ellipse has two unequal diameters or axes, which are at right angles to each other. The longer one is called the transverse diameter, and the shorter one the conjugate diameter.

The transverse diameter is also called the major axis, as AB (Fig. 219), and the conjugate diameter the minor axis, as CD.
155. The two axes AB and CD being given, to construct an ellipse.
Take a strip of paper and set off upon it the distance FH , equal to half the major axis and the distance FG , equal to half the minor axis. By keeping the point $G$ on the major axis, and the point H on the minor axis, the point $F$ will give a point in the ellipse. A succession of points can be


Fig. 219. found in this manner, through which draw a fair curve, which will be the required ellipse.
156. To construct an ellipse, given an axis and two foci.

An ellipse has two foci, as the points A and B, Fig. 220, and the sum of the radii from these two points is always equal.

Let A and B represent two pins, and ABC a piece of thread. The point of a pencil is placed inside the thread at C and moved so as to keep the thread always tight; the point will trace out an ellipse. As the length of the thread is constant, the sum of the two radii is constant also.

The length of the major or minor axis given will determine the length of the thread.
157. To construct an ellipse by means of intersecting lines, the transverse diameter AB and the conjugate diameter CD being given.
Draw the lines $A B$ and $C D$ bisecting each other at right angles in the point E (Prob. 5). I)raw HK and FG parallel to AB , through C and D , and HF and KG parallel to CD through A and B . (Prob. 3). Divide AH and BK into any number of equal parts, say four (Prob. 9), and AE and EB into the same number. Join C with the three points in AH and BK, and produce lines from $D$ through the three points in AE and


Fig. 22 I. EB. Where these lines intersect those drawn from C, points in one-half of the ellipse will be obtained. Find corresponding points for the other half in the same manner, and draw a fair curve through the points obtained, which will be the required ellipse.
158. To find the normal and tangent to a given ellipse ABCD , at a given point $P$.


Fig. 222.

With $C$ as centre, and radius equal to EA , draw the arc FH , which will give the two foci in F and H. Join the given point $P$ with $F$ and $H$, and bisect the angle FPH by the line PK (Prob. 12). PK is the normal. Draw the line NO through $P$, perpendicular to PK . This is the tangent required.
159. To complete an ellipse from an elliptical curve.

Let AB be the given curve. Draw any two sets of parallel chords and bisect them (Prob. I). Join the points of bisection in each set by lines meeting in C. Produce one of these lines till it meets the given curve in D. With $C$ as centre, and CD as radius, set off on the given curve the point A. Join AD. Through C draw a line HK parallel to AD (Prob. 3), also the line CL perpendicular to $A D$ (Prob. 7). Produce CL to M , making CM equal to CL. Also make CK equal to CH .


Fig. 223. Then LM will be the transverse and HK the conjugate diameters. The ellipse can then be completed by any of the constructions already described. From A and D lines are drawn parallel to LM ; with C as centre, and radius CD , set off E and $F$, these will be two more points in the ellipse.
160. To draw an ellipse to pass through three given points $\mathrm{A}, \mathrm{B}, \mathrm{C}$.

Join AC and bisect it in D. Join BD. From A and C draw the lines AE and CF parallel to BD . Through B draw the line EF parallel to AC. Produce BD to H , and make DH equal to BD . Divide AD and DC and also AE and CF into a number of equal parts, say four. Join the divisions on AE and CF to B . From


Fig. ${ }^{224 .}$
$H$, through the divisions on $A C$, draw lines till they meet the corresponding lines drawn to $B$. Draw a fair curve through these points, which will give half of the required ellipse. Proceed in the same way with the other half.
161. To determine points for drawing a parabola, the focus A and the directrix BC being given.
Draw the line EAD perpendicular to the directrix BC (Prob. 7), which will give the axis. Bisect AD in F (Prob. I), which will be the vertex of the curve. Take any points $a, b, c, d$, and $e$ in the axis, and draw perpendiculars through them. From A as centre, mark off on the perpendiculars, arcs with radii equal to $a \mathrm{D}, b \mathrm{D}, c \mathrm{D}, d \mathrm{D}$, and $e \mathrm{D}$, cutting the perpendiculars in $a^{\prime}, b^{\prime}, c^{\prime}$, $d^{\prime}$, and $e^{\prime}$. These are the points required for the lower half of the parabola. The points above the axis are found in the same manner.


Fig. 225.
162. To draw a tangent to a parabola at a given point $H$.

Join AH. From A set off AK on the axis produced equal to AH. Join KH, which will be the required tangent. This could also be found by drawing a line from $H$ parallel to the axis till it meets the directrix in $B$, and then bisecting the angle AHB by the line KH (Prob. 12), which is the tangent. If from H we draw the line HL perpendicular to the tangent, it will be the normal.
163. To draw a parabola, an abscissa AB and an ordinate BC being given.
Complete the rectangle ABCD . Divide BC into any number of equal parts, say six (Prob. 9), and CD into the same number. From each division in BC draw lines parallel to CD (Prob. 3), and from each of the divisions in CD draw lines to the vertex $A$. Where these lines of corresponding numbers intersect, e.g. where I intersects with $\mathrm{I}^{\prime}, 2$ with $2^{\prime}$, etc., are points in the parabola. Find corre-


Fig. 226. sponding points on the opposite side of the axis, and draw a fair curve through them.
164. To draw an hyperbola, the diameter AB , an ordinate CD , and an abscissa BD being given.
Draw BE parallel to CD (Prob. 3), and complete the rectangle. Produce BD, and make $A B$ equal to the given diameter. Divide CD and CE into any number or equal parts, say four (Prob. 9), a, b, c. The divisions on CD join to A, and those on CE to B. c The intersection of the corresponding lines, e.g. where $a$ intersects $a^{\prime}, b b^{\prime}$, and $c c^{\prime}$, are points in the hyperbola required. Find corresponding points for


Fig. 227. the other half, and draw a fair curve through them.

A form of hyperbola frequently used is the rectangular hyper-


Fig. 228. bola. Let AB and AC represent two axes, and $E$ the vertex of the curve. Complete the rectangle ABDC. Take any point H in CD and join it to A. Let fall a perpendicular from E till it meets HA in O (Prob. 7). From O draw OK parallel to AB till it meets a line from H parallel to AC in the point K (Prob. 3). This will be one point in the curve, and others may be found by taking fresh points on CD and treating them in a similar manner.

One peculiar property of this figure is that, if we take any point in the curve and draw lines from it perpendicular to the lines AB and AC ,-for example, KN and KM ,-the rectangle contained by the two lines is always equal, i.e. $\mathrm{KN} \times \mathrm{KM}$ would be the same for any point in the curve.
165. A mechanical method of drawing a parabola or hyperbola.

Let $A B$ represent the edge of a drawing-board and $C D$ the


Fig. 229. edge of a tee-square. Take a piece of string equal in length to CD, fix one end at D and the other at E , which is the focus of the curve. If a pencil be held against the string so as to keep it tight against the tee-square when the tee-square is moved upwards, the pencil will trace half a parabola. AB is the directrix, and K the vertex of the curve. Compare this method with the construction of Prob. 16I.

If the angle DCA were an acute or obtuse angle instead of a right angle, the pencil would trace an hyperbola.
166. To draw an oval by arcs of circles, its transverse diameter $A B$ and its conjugate CD being given.
Set off on $A B$ the distance $A E$ equal to half the conjugate diameter. Through $E$ draw the line $F G$ perpendicular to AB (Prob. 7). With E as centre and EA as radius, draw the semicircle CAD. From C and D set off CF and DG equal to EA. From B set off BH equal to half of EA. Join FH and GH. With F and $G$ as centres, and FD, GC as radius, draw the arcs DL and CK. With H as centre, and HK as radius, draw the arc


Fig. 230. KBL, which wilt complete the oval required.

The conic sections are of frequent occurrence both in science and art: the heavenly bodies trace them in their courses; they are used by engineers where great strength is required, such as the construction of bridges ; and they form the contour of mouldings, etc. Those subtle curves that we admire in the outline of Japanesehandscreens and vases are often parabolas.

Fig. 23I is an illustration showing how these curves are applied to art forms.

To draw the curve $C B$. Draw the lines AB and AC at right angles to each other (Prob. 5). Divide each into the same number of equal parts (Prob. 9), and join them. Proceed in the same manner with the other curves.s.

## Cycloidal Curves.

If a circle is rolled along a line in the same plane, a point in the circle will describe a curve of a class called cycloidal.

The line along which the circle rolls is called a director, and the point itself is called the generator.

The curve is called a Cycloid when the generator point is in the circumference of the rolling circle and the director is a straight line ; but a Trochoid when the point is not in the circumference of the circle.

When the director is not a straight line, but the outside of another circle, and the generator is in the circumference of the rolling circle, the curve described is called an Epicycloid; but when the point, or generator, is not in the circumference of the rolling circle, it is called an Epitrochoid.

If the director is the inside of a circle and the generator a point in the circumference of the rolling circle, the curve is called a Hypocycloid ; but if the generator is not in the circumference of the rolling circle, it is called a Hypotrochoid.

The Epicycloid and Hypooycloid are the true curves for the teeth of gearing. The director is the pitch circle of each wheel : and if the rolling circle be the same for the whole set, they will gear into one another.

In constructing a cycloid it is necessary to make a line equal in length to the arc of a semicircle. The exact relation betwep ${ }^{-1}$ the diameter and circumference of a circle cannot be express in numbers ; but the following problem will enable us to arriat an approximation, correct to six places of decimals.

## 167. To draw a line equal to the length of a semicircle.

Let AC represent the radius. Draw the semicircle ABD. Produce AC to D, and draw BC perpendicular to it. From A and D draw tangents parallel to BC , and through B draw a tangent parallel toto AD . From B set off BE equal to the radius, and draw the line BF through En Produce the tangent through A to $H$, and make AH equal to AD. Join HF, which will be the line required

If we take AC to represent a length equal to the diameter of a circle, then HF will equal the circumference.


Fig. 232.
We are also enabied to find the length of an arc by this means; e.g. the arc to the chord formed by one side of a pentagon. - IfFH is eyal tn the circumference, then $\frac{\mathrm{FH}}{r_{0}}=$ the lenoth $n^{f}$ titue arc required.

## 168. To draw a cycloid.

Let $A B$ be the director, and $p$ the generator or point in the rolling circle AMp . Draw $\mathrm{A} k$ equal in length to half the circumference of the circle $A M p$, and divide it into any number .? equal divisions (Prob. 10), say six, at $d, e, f, g$, and $h$. Divide the semicircle into the same number of equal divisions (Prob. 14), and draw lines from each division parallel to the director AB . Draw the line CK from the centre of the circle parallel to AB . Draw lines perpendicular to AB at the points $d, e, f, g, h$, and $k$ till they meet the line CK in the points $\mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}$, and K. With each of the points D, E, F, G, H, and K as intres, and a radius equal to $\mathrm{C} p$, draw arcs cutting the parallel $e^{\prime}$ drawn from the divisions in the semicircle in the points in $\cap, P, Q, R$, and $T$. This will give points in half the cycloid. nd the corresponditig points for the remaining half, and draw fair curve through the points, whiger will give the cycloid fuired.

To determine the tangent and normal to the curve at any point $t$ :-Draw the line $t t^{\prime}$ parallel to AB till it meets the


Fig. 233.
generating circle at $t^{\prime}$. Join $t^{\prime} \mathrm{A}$. Through $t$ draw the line WW' parallel to $t^{\prime} \mathrm{A}$. This will be the normal to the curve. The tangent $r s$ is at right angles to this line.

## 169. To draw an epicycloid.

Note.-The length of the director für a complete curve is to the whole circle as the radius of the rolling circle is to the radius of the director; e.g. if radius of rolling circle $=\mathrm{I}$ inch, and that of director $=6$ inches ; then the director $=\frac{1}{6}$ of the circle.

Let $A B$ be the director, which is a part of a circle, and $\phi$ the generator. Take Ak equal in length to half the rolling circle, AM $p$, and divide it into any number of equal divisions, say six, at $d, e, f, g$, and $h$. Divide the semicircle into the same number of equal divisions, and draw lines from these points, as well as from the centre of the circle, concentric with the arc AB. From the centre of the circle that contains the arc AB draw lines through the points $d, e, f, g, h$, and $k$ till they meet the arc drawn from the centre of the rolling circle. With D, E, F, G, H , and K as centres, and a radius equal to $\mathrm{C} \rho$, draw arcs till they meet the concentric arcs drawn from the divisions of the e
 sponding points fom the opppsithersidfx and thawsa fair ot av: e through all the points whicha widh benthe er ifginioud Eequancedf

At any point t to statwan tangent and normaljtguthe of urve, proceed as follows. Driw the arc $t t^{\prime}$ concentric to the ary
till it meets the generating circle in $t^{\prime}$. Join $t^{\prime} \mathrm{A}$. With $t$ as centre, and radius equal to $t^{\prime} \mathrm{A}$, draw an arc intersecting AB at


Fig. 234.
' $w$ '. Join $t w$ ', and produce it to $w$. This is the normal. The tangent $r s$ is at right angles to it.

## 170. To draw a hypocycloid.

Let $A B$ be the director, which is the arc of a circle, and $p$ the generator, which is a point in the circumference of the rolling circle M. Make $\mathrm{A} \not \approx$ equal in length to half of the circle M , and divide it into any number of equal parts, say six, at $d$, $e, f, g$, and $h$. Divide the semicircle into the same number of equal parts, and from the centre of the circle containing the arc AB draw concentric arcs from these points, as well as from the centr e C. Draw lines from the points $d, e, f, g, h$, and $k$ towards the cenire of the circle containing the arc AB till they meet the arc from the centre C in the points $\mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}$, and K ./ With os noints as centres, and a radius equal to $\mathrm{C} p$, draw ircs till they meet the concentric arcs drawn from the divisions r the semicircle in the points $\mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}$, and T . Find ic corresponding points for the other half, and draw a fair is e through all the prints, which will give the hypocycloid quired.

The tangent and normal at any point $t$ are thus 0 .. ed. Draw the arc $t t^{\prime}$ concentric to the arc AB till it meets the generating circle at $t^{\prime}$. Join $t^{\prime} \mathrm{A}$. With $t$ as centre, and radius


Fig. 235 .
equal to $t^{\prime} \mathrm{A}$, set off on AB the point $w^{\prime}$. Join $t w w^{\prime}$, and produce it to $w$. This is the normal. The tangent $r s$ is at right angles to it.
171. To construct a continuous curve, by a combination of arcs of different radii, through a number of given points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, $\mathrm{F}, \mathrm{G}$, and H .

Join the points $\mathrm{AC}, \mathrm{CD}, \mathrm{DE}$, etc. Find the centre K of the


Fig. ${ }^{236}$. circle containing the arc ABC (Prob. 35). Join CK. Bisect the line $C D$ at right angles, and produce the bisecting perpendicular till it meets CK produced in L. Join DL. Bisect the line DE and produce the visecting perpendicular till it meets DL produced in :r. Find the remaining points $\mathrm{N}, \mathrm{O}$, and $P$ in the same manner. The points $\mathrm{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}$, and $P$ are the centres of the circles containing the arcs necessary for joining the given points.

## Cifid To construct an Archimedean spiral of one revolution.

I) raw a circle and divide it by radii into any number of equal parts, say tweive (Prob. 9) $a, b$, $c, d$, etc. Divide the radius $o$ into a corresponding number of equal parts $1,2,3,4$, etc. (Prob. 9). From the centre of the circle, with radius I, draw an arc till it meets the radius $a$ in A , and from 2 till it meets the radius $b$ in B , and so on till the whole twelve are completed. Draw a fair curve through these points, A, B, C, D, etc., which will give the spiral required and


Fig. 237. proceed in exactly the same manner for further revolutions.
173. To draw the logarithmic spiral.

The logarithmic spiral was discovered by Descartes. It is also called the equiangular spiral, because the angle the curve makes with the radius vector is constant. The curve also bears a constant proportion to the length of the radius vector

Take any line AC for the radius vector, and bisect it in D (Fig. 238). With D as centre, and radius DA, draw the semicircle ABC . From the points $A$ and $C$ draw any two lines $A B$ and CB cutting the semicircle. Then ABC is a right-angled triangle.

Bisect the line $B C$ in $E$. With E as centre, and EB as -adius, draw the semicircle


Fig. 238.

BFC . Make the angle BCF equal to the angle ACB , and produce the line till it meets the semicircle in F . Join BF . The triangle BFC is then similar to the triangle ABC . By repeating this construction we obtain a succession of similar triangles radiating from a common centre $C$, and all forming equal angles at this point. The exterior points of these triangles, viz. $\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{M}, \mathrm{N}, \mathrm{O}$, etc., are points in the required spiral.

As each triangle with its curve forms a similar figure, it is evident that the curve must form a constant angle with its radius vector, i.e. the line radiating from C , and the portion of the curve accompanying each triangle, must also bear a constant proportion to the length of its radius vector.

If we bisect the angle $A B F$ by the dotted line $H B$, this line will be the normal to the curve ; and the line KL, being drawn at right angles to HB , is the tangent to the spiral.

As all the angles at $C$ are equal, the spiral could be constructed with greater facility by


Fig. 239. first drawing a circle and dividing it into an equal number of parts by radii, as shown in Fig. 239.

Let AC be the radius vector. Bisect it in D. With D as centre, and radius DA , draw an arc cutting the next radius CB in $B$. Proceed in the same manner with each radius in succession, which will determine the points $\mathrm{H}, \mathrm{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}$, P , etc. Draw a fair curve through these points, and we shall obtain a logarithmic spiral.

The greater the number of radii used in the construction, the larger will be the angle BAC ; but the angle ABC will always be a right angle, as will be seen by the construction in Fig. 238.
174. To draw a spiral adapted for the Ionic volute by means of arcs.

Divide the given height AB into eight equal parts (Prob. 9). Bisect the fourth part in the point C (Prob. I), and from it
draw a line perpendicular to AB (Prob. 7). Make this line equal in length to four of the divisions of $A B$, which will give the eye of the volute D . This is shown to a larger scale at E. With D as centre, draw a circle with a radius equal to $\mathrm{C}_{4}$. Inscribe a square in this circle (Prob. 56), and bisect each of its sides in the points $1,2,3$, and 4 (Prob. 1). Join these points, and draw diagonals. Divide each semi-diagonal into three equal parts and join them (Prob. 9), thus making three complete squares parallel to each other. The corner of each of these squares in succession will be the centre of one of the arcs, commencing at I , with L as radius, as shown by dotted lines and arrow-heads.

## EXERCISES.

1. Construct an ellipse: major axis $3.75^{\prime \prime}$, minor axis $2.25^{\prime \prime}$. Select any point in the curve, and draw a tangent to it.
2. Construct an ellipse ; the foci to be $2 \frac{1}{4}^{\prime \prime}$ apart, and the transverse diameter $3 \frac{3^{\prime \prime}}{\frac{\prime 2}{2}}$.
3. Draw a rectangle $3.25^{\prime \prime} \times 2.3^{\prime \prime}$, and inscribe an ellipse within it.
4. Draw a parallelogram $3 \frac{7^{\prime \prime}}{3} \times 2 \frac{3^{\prime \prime}}{3}$, two of its angles to be $60^{\circ}$; inscribe an ellipse within it.
5. Construct an ellipse by means of a paper trammel ; the transverse diameter being $4 \frac{1^{\prime \prime}}{}$, and the conjugate diameter $3^{\prime \prime}$ (Prob. 155).
6. Draw the two diameters of an ellipse each $3^{\prime \prime}$ long, and at an angle of $45^{\circ}$ with each other ; complete the ellipse.
7. Make a tracing of the ellipse given in question 3 , and find the diameters, foci, tangent, and normal.
8. With an abscissa $3^{\prime \prime}$ long and an ordinate $2^{\prime \prime}$ long, construct a parabola.
9. With a diameter I. $4^{\prime \prime}$, an ordinate I. $8^{\prime \prime}$, and an abscissa 1.4", construct a hyperbola.
10. Draw a rec'angle $3^{\prime \prime} \times 2^{\prime \prime}$, and let two adjacent sides represent the axes of a rectangular hyperbola; measure off along one of its longer edges $\frac{1^{\prime \prime}}{2}$, and let this point represent the vertex of the curve ; complete the hyperbola.
11. Draw a line $4^{\prime \prime}$ long to represent an abscissa of a parabola; at one end draw a line $3^{\prime \prime}$ long, at right angles to it, to represent the directrix ; from the directrix, along the abscissa, set off $\mathrm{I}^{\prime \prime}$ to mark the focus; complete the parabola. At any point in the curve, draw a tangent and normal to it.
12. Draw geometrically an ellipse, a parabola, a hyperbola, each 2 inches long, and a cone 2 inches high, and write the name to each. Show the following five sections on the cone-a horizontal section, a vertical one, not through the apex, and one through the apex, onc parallel to one side, and one cutting both inclined sides. Name figure each section makes.
(May, '96.)
13. The foci of an ellipse are $2 \frac{1}{2}^{\prime \prime}$ apart and its major axis is $3 \frac{12^{\prime \prime}}{}$ long. Describe half the curve.
(A pril, '96.)
14. The foci of an ellipse are $2 \frac{1}{2}^{\prime \prime}$ apart, and its minor axis is $2^{\prime \prime}$ long. Draw the curve, and draw also a tangent from a point on the curve $\mathbf{r}^{\prime \prime}$ from one of the foci.
(June, 'oo.)
15. Draw the curve (Fig. 241) every point of which is at equal distances from the line $P Q$ and the point $F$. The curve, which is a parabola, need not be shown below the line ST.
(April, '96.)


Dimensions to be trebled. Fig. ${ }^{241}$.
16. Draw a straight line $\mathrm{AB} 3^{\prime \prime}$ long. Bisect AB in F . At F draw FV $\frac{3^{\prime \prime}}{4}$ long at right angles to AB. F is the focus, and V the vertex of a parabola, A and B being points on the curve. Draw the curve from A to B, showing the construction for at least 4 points. (June, 'oo.)
17. Two conjugate diameters of an ellipse are $3 \frac{1}{2}^{\prime \prime}$ and $2 \frac{1}{2}^{\prime \prime}$ long respectively, and cross one another at an angle of $60^{\circ}$. Draw the curve. (April, '99.)
18. An arch in the form of a semi-ellipse is $6^{\prime}$ wide and $2^{\prime}$ high. Describe the curve, and draw two lines perpendicular to it from two points on the curve, each $2^{\prime}$ from the top point of the arch.

Scale (which need not be drawn) $\frac{1_{2}^{\prime \prime}}{}$ to $\mathbf{I}^{\prime}$.
(April, '98.)
19. Draw the arch opening shown (Fig. 242), using the figured dimensions. The curve is a semi-ellipse, of which I'Q is a diameter, and RS is half its conjugate character.
(June, '99.)
20. Describe the spiral of Archimedes of three revolutions, whose radius is 2 inches.
(April, '98.)
21. Within a circle of 2 " radius describe a 'spiral of Archimedes' of one revolution.
(June, '99.)
22. Sketch the three sorts of spiral, and explain how each is generated, and illustrate each sort by shells, plants, or animals. (May, '96.)

## PART II.

## SOLID GEOMETRY.

## CHAPTER XII.

INTRODUCTION.
In the preceding subject, Plane Geometry, we have been restricted to figures having length and breadth only, but Solid Geometry treats of figures that have thickness in addition to length and breadth.

The objects taken to illustrate the principle of this subject are described under the head of Definitions, Solids (page io).

By means of Practical Solid Geometry we are enabled to represent on a plane-such as a sheet of paper-solid objects in various positions, with their relative proportions, to a given scale.


Fig. 243 .

Let us take some familiar object, a dressing-case for instance, ABDC, Fig. 243 , and having procured a stiff piece of drawing-paper HKLM, fold it in a line at X , parallel to one of its edges; then open it at a right angle, so that HX will represent the edge of a vertical plane, and XL the edge of a horizontal plane; the line at $X$, where the two planes intersect, is called the line of intersection, intersecting line, or ground line; it shows where
the two planes intersect each other, and is generally expressed by the letters X and Y , one at each end.

Having placed the dressing-case on the horizontal plane, with its back parallel to the vertical plane, let us take a pencil and trace its position on the horizontal plane by drawing a line along its lower edges; also its shape on the vertical plane. This can be done by placing the eye directly opposite each of its front corners in succession and marking their apparent position on the vertical plane, and joining them. Having done this, we will remove the dress-ing-case and spread the paper out flat upon a table: this is


Fig. 244. shown in Fig. 244. We have now two distinct views of the object. The lower one is called a PLAN, and represents the space covered by the object on the horizontal plane, or a view of the dressing-case seen from above. The upper view shows the space covered on the vertical plane, and is called an elevation : it represents the front view of the object.

In Solid Geometry all objects are represented as they would appear traced or projected on these two planes at right angles to each other: they are called co-ordinate planes. It is not necessary that the object should be parallel to them, as in Fig. 243: we can arrange it in any position, making any possible angle with $\bar{z}$ either plane, but the line connecting the point on the object with its respec-


Fig. 245. tive plane must always be perpendicular to that plane. We shall understand this better if we refer to Fig. 245, in which we will imagine the dressing-case suspended in mid-air, with its back
still parallel to the vertical plane, but its under side inclined to the horizontal plane. We will now trace it on each plane as before, then by spreading the paper out flat we get a drawing as shown in Fig. 246.

The student should compare Fig. 243 with Fig. 244, as well as Fig. 245 with Fig. 246, so as to thoroughly understand the relation between the co-ordinate planes.

The lines $\mathrm{A} a^{\prime}, \mathrm{B} b^{\prime}, \mathrm{E} e$ (Fig. 246), are all perpendicular to the vertical plane ; and the lines $\mathrm{A} a, \mathrm{~B} b, \mathrm{C} c$, etc., are perpendicular


Fig. 246. to the horizontal plane. These lines are called projectors, and are here represented by dotted lines. The points in which these lines meet the two coordinate planes are called projections: if they are on the vertical plane they are called vertical projections, and if on the horizontal plane horizontal projections, of the different points ; e.g. $a^{\prime}$ is the vertical projection of point A, and $a$ is its horizontal projection. The length of the horizontal projector shows the distance of the point from the vertical plane, and the length of the vertical projector its distance from the horizontal plane.

This method of representing solid objects by projection on two planes is called orthographic projection, and is described more in detail in Chapter XIV. As the projectors are parallel to each other, it may also be called parallel projection.

All through this subject the points of the object are distinguished by capital letters, as A, B, etc., while their horizontal projections are represented as $a, b$, etc., and their vertical pro jections as $a^{\prime}, b^{\prime}$, etc. ; by this means we are enabled to distinguish the plan from the elevation. V.P. will also be used to express the vertical plane, and H.P. the horizontal plane; the letters XY will always stand for the ground line.

The student should take particular notice that the lower points in the plan always represent the front points in the elevation.

It is not necessary to have an object to trace ; if we know its dimensions, and its distances from the two planes, we can construct the plan and elevation as shown in Figs. 244 and 246.

Figs. 243 and 245 are perspective views, and Figs. 244 and 246 are geometrical drawings of the same object. If the latter were drawn to scale, we could find out the length, breadth, and thickness of the object from these drawings.

Each perspective view is supposed to be taken from one fixed point, i.e. the eye ; and lines drawn from different parts of the object converge towards the eye considered as a point. These lines represent rays of light from the object, and are called visual rays : they form a cone, the vertex of which is the position of the eye ; consequently, Perspective is called conical, radial, or natural projection, because it represents objects as they appear in nature. It is impossible to see an object as it is represented by orthographic projection.

We will now take four simple solids, viz. a cube, a rectangular solid, a pyramid, and a triangular prism, and show the different positions they can occupy with reference to the co-ordinate planes, i.e. the V.P. and H.P.

Fig. 247 represents the four solids in what is called simple positions, i.e. parallel to both the V.P. and H.P.


Fig. 247.

A is the plan of the cube

| B | $\because$ |
| :--- | :--- |
| C | $"$ |
| D | $\because$ |

99
rectangular solid pyramid triangular prism
and $\mathrm{A}^{\prime}$ its elevation.

| $B^{\prime}$ | $\neq$ |
| :--- | :--- |
| $C^{\prime}$ | $"$ |
| $D^{\prime}$ | $\because$ |



Fig. 248.
Fig. 248 represents the same solids with their bases on the H.P. as before, but their sides are now inclined to the V.P.


Fig. 249.
Fig. 249 shows them with their fronts and backs, parallel to the V.P., as in Fig. 247, but with their bases inclined to the H.P.


Fig. 250.

Fig. 250.--They are here represented inclined to both the V.P. and H.P., but they still have one set of edges parallel to the H.P.

Fig. 25I.-Here they are shown with every line inclined to both planes : instead of having one edge resting on the H.P., as in


Fig. 251.
Fig. 250, they are each poised on a corner. To distinguish this position from the one illustrated in Fig. 250, we will call it compound oblique ; although Figs. 250 and 251 generally come under one head, as objects inclined to both planes

## CHAPTER XIII.

## SIMPLE SOLIDS IN GIVEN POSITIONS TO SCALE.

Note.-Feet are indicated by one dash, and inches by two dashes : thus- $3^{\prime} 6^{\prime \prime}$ represents 3 feet 6 inches.

The student should draw the problems in Solid Geometry to a scale three times that of these figures.
175. To project a quadrilateral prism $5^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime} \times 2 \frac{1_{2}^{\prime \prime}}{}$ with one of its smaller faces on the H.P., parallel to the V.P., and $\frac{3^{\prime \prime}}{4}$ from it. Scale $\frac{1}{6}$ full size. Fig. 252 A .
First draw the line XY; then draw the plan abcd $\frac{3}{4}$ inch below it. Draw perpendicular lines above XY, immediately over the points $a$ and $b$, $5^{\prime \prime}$ in height, which give the points $a^{\prime}$ and $b^{\prime}$. Join $a^{\prime} b^{\prime}$. This is the elevation of the solid.
176. To project the same solid with one of its longer faces resting on the H.P., parallel to the V.P., and $1^{\frac{3}{4}}$ from it, to the same scale. Fig. 252 B.
Draw the plan efgh, $5^{\prime \prime} \times 2 \frac{11^{\prime \prime}}{2}$, and $1 \frac{3}{4}$ inches below XY. Draw perpendiculars above XY, $2 \frac{1}{2}{ }^{\prime \prime}$ high, and directly over the points $e$ and $f$, which will give the points $e^{\prime}$ and $f^{\prime}$. Join $e^{\prime}$ and $f^{\prime}$, which completes the elevation.

The student should now project the four solids illustrated in Fig. 247 in the positions there shown, but to the following dimensions and scale :

A to have a base $4^{\prime \prime} \times 4^{\prime \prime}$, to be $4^{\prime \prime}$ high and $2 \frac{1}{2}{ }^{\prime \prime}$ from V.P.

| $8^{\prime \prime} \times 4^{\prime \prime}$ | , | $2^{\prime \prime}$ | , | $2 \frac{1^{\prime \prime}}{2}$ | , | V.P. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $4^{\prime \prime} \times 4^{\prime \prime}$ | $"$ | $8^{\prime \prime}$ | $"$ | $2 \frac{\frac{1}{2}^{\prime \prime}}{}$ | $"$ | V.P. |
| $6^{\prime \prime} \times 4^{\prime \prime}$ | $"$ | $4^{\prime \prime}$ | $"$ | $1 \frac{1^{\prime \prime}}{2}$ | $"$ | V.P. |

Scale $\frac{1}{4}$ full size.
177. To project a quadrilateral prism $10^{\prime \prime} \times 5^{\prime \prime} \times 5^{\prime \prime}$, with one of its smaller faces on the H.P., at an angle of $45^{\circ}$ with V.P., and one edge $3 \frac{l^{\prime \prime}}{\frac{1}{\prime}}$ from V.P. Scale $\frac{1}{12}$ full size. Fig. 253 A.

Draw XY. Take the point $d, 34^{1 \prime}$ below XY, and draw the square abcd below this point with its sides at an angle of $45^{\circ}$ with XY. This will be the plan. Erect perpendiculars 10 " high above XY , and directly over the points $a$, $b$, and $c$. Join the tops of these per-
 pendiculars, which completes the elevation.
178. To project the same solid lying on one of its longer faces on the H.P. with its longer edges forming an angle of $30^{\circ}$ with the V.P., and one of its corners $1_{4}^{3^{\prime \prime}}$ from V.P., to the same scale. Fig. 253 B.

Draw the point $h, 1 \frac{3}{4}^{\prime \prime}$ below XY, and construct the plan efg $/ h$ at the required angle below this point. Erect perpendiculars $5^{\prime \prime}$ high above XY, immediately over the points $g$, $e$, and $f$. Join the tops of these perpendiculars, which conipletes the elevation.

The student should now project the four solids illustrated in Fig. 248 , in the positions there shown, but to the following dimensions and scale :
A to have a base $5^{\prime \prime} \times 5^{\prime \prime}$, to be $5^{\prime \prime}$ high, with one side inclined at an angle of $30^{\circ}$ with the V.P., and $2^{\prime \prime}$ from it.
B to have a base $10^{\prime \prime} \times 5^{\prime \prime}$, to be $2 \frac{1_{2}^{\prime \prime}}{}$ high, with both sides inclined at an angle of $45^{\circ}$ with the V.P., and $2^{\prime \prime}$ from it.
C to have a base $5^{\prime \prime} \times 5^{\prime \prime}$, to be $10^{\prime \prime}$ high, with one edge of the base making an angle of $60^{\circ}$ with the V.P., and its nearest point $2^{\prime \prime}$ from it.
D to have a base $\delta^{\prime \prime} \times 5^{\prime \prime}$, to be $5^{\prime \prime}$ high, with both sides of its base making an angle of $45^{\circ}$ with the V.P., and the nearest point $2^{\prime \prime}$ from it.

Scale $\frac{1}{3}$ full size.
179. To project a quadrilateral prism $7 \frac{1}{2}{ }^{\prime \prime} \times 3 \frac{3^{\prime \prime}}{4} \times 3 \frac{3^{\prime \prime}}{4}$, resting on one of its shorter edges on the H.P., and with its longer edges parallel to the V.P., but inclined at an angle of $60^{\circ}$ to the H.P.; one of its faces to be $2 \frac{11^{\prime \prime}}{}$ from V.P. Scale $\frac{1}{9}$ full size. Fig. 254 B.


Draw XY. At point $e^{\prime}$ draw the elevation $e^{\prime} a^{\prime} b^{\prime} c^{\prime}$ at the required angle. $2 \frac{1^{\prime \prime}}{}$ below XY draw the line $d f$ parallel to it.

Let fall lines from $a^{\prime}, b^{\prime}$, and $c^{\prime}$, at right angles to XY, and make $d a$ and $f c$ each $33^{\prime \prime}$ long. Join $a c$, which completes the plan.

The student should now project the four solids illustrated in Fig. 249 from the following conditions :
A to be $2 \frac{1}{2}^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime}$, with its base inclined at an angle of $45^{\circ}$ to H.P. ; to be parallel to the V.P., and $2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ from it.
B to be $5^{\prime \prime} \times 2 \frac{1_{2}^{\prime \prime}}{2} \times \mathrm{I}_{\frac{1}{4}}$, with its base inclined at an angle of $30^{\circ}$ to H.P. ; to be parallel to the V.P., and $2 \frac{1}{2}{ }^{\prime \prime}$ from it.
C to be $2 \frac{1}{2}^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime} \times 5^{\prime \prime}$, with its base inclined at an angle of $45^{\circ}$ to H.P. ; to have the edge of its base parallel to the V.P., and $2 \frac{1}{2}^{\prime \prime}$ from it.
D to be $4^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime} \times 2 \frac{1}{2}^{\prime \prime}$, with its base inclined at an angle of $30^{\circ}$ to H.P. ; to have its ends parallel to V.P., and $I^{\frac{3}{4}}{ }^{\prime \prime}$ from it. Scale $\frac{1}{2}$ full size.
Before proceeding with the next problem, Fig. 254 B, it will be necessary to understand thoroughly the angles which the solid forms with the co-ordinate planes.

The longer edges are still inclined to the H.P. at an angle of $60^{\circ}$; but instead of being parallel to the V.P., as in Fig. 254 A, they are in planes inclined to the V.P. at an angle of $45^{\circ}$. This does not mean that they form an angle of $45^{\circ}$ with the V.P. Let us illustrate this with a model.

Take a sheet of notepaper and draw a diagonal to each of its inside pages, as $a b$ and $b c$, Fig. 255. Now stand it on a table against the $45^{\circ}$ angle of a set-square. The two pages will then represent two planes at an angle of $45^{\circ}$ with each other. Let the page a represent the V.P.,


Fig. 255. and the line $b c$ one of the edges of the solid. The angle which $b c$ makes with the page $a$ will be considerably less than $45^{\circ}$.
180. To project a quadrilateral prism $7 \frac{11^{\prime \prime}}{2} \times 3_{4}^{\frac{31}{4}} \times 3 \frac{3^{\prime \prime}}{4}$, resting on one of its shorter edges on the H.P., with its longer edges inclined to the H.P. at an angle of $60^{\circ}$, and in vertical planes inclined to the V.P. at an angle of $45^{\circ}$; one of its lower corners to be $1_{4}{ }^{\prime \prime}$ from V.P. Scale $\frac{1}{9}$ full size. Fig. 254 B.
Find the position of point $n, \mathrm{I}^{\frac{1}{4}}$ below XY, and draw the lines $n m$ and $n k$ at an angle of $45^{\circ}$ to XY. Make $n m$ and $n k$ equal in length to $d f$ and $d a$ (Fig. 254 A). Draw the line $k l$ parallel to $m m$, and of the same length. Join $l m$. Make $k g$ and $l o$ equal to $a e$ and $c b$ (Fig. 254A). Draw $g h$ and $o p$ perpendicular to $k l$. This will complete the plan. As every point in the elevation is found in precisely the same way, it is only necessary to explain the projection of one point : $o^{\prime}$, for example.

Draw a perpendicular from $o$ on plan till it meets a horizontal line drawn from $b^{\prime}$ (Fig. 254 A ). This gives the position of point $o^{\prime}$.

The student should now project the four solids shown in Fig. 250 from the following conditions :
A to be $4^{\prime \prime} \times 4^{\prime \prime} \times 4^{\prime \prime}$, with one set of edges parallel to H.P. ; its other edges to be inclined to H.P. at an angle of $45^{\circ}$, and in vertical planes inclined to V.P at an angle of $30^{\circ}$; its nearest corner to be $4^{\prime \prime}$ from V.P.
B to be $8^{\prime \prime} \times 4^{\prime \prime} \times 2^{\prime \prime}$, with one set of edges parallel to H.P. ; its longest edges inclined at an angle of $30^{\circ}$ to H.P., but which, with its shortest edges, are to be in vertical planes inclined to the V.P. at an angle of $30^{\circ}$. Its nearest corner to be $4^{\prime \prime}$ from V.P.
C to be $4^{\prime \prime} \times 4^{\prime \prime} \times 8^{\prime \prime}$; its base to be inclined to the H.P. at an angle of $45^{\circ}$; its axis in a vertical plane inclined to V.P. at an angle of $30^{\circ}$; and its nearest corner $4^{\prime \prime}$ from V.P.
Note.-The axis is a line drawn from the vertex to the centre of the base ; as $a b$, Fig. 250.
D to be $6^{\prime \prime} \times 4^{\prime \prime} \times 4^{\prime \prime}$, to have one side inclined at an angle of $30^{\circ}$ with H.P., and its ends in a vertical plane inclined to V.P. at an angle of $60^{\circ}$. The nearest corner to be $4^{\prime \prime}$ from V.P. Scale $\frac{1}{4}$ full size.
Note.-The heights in these elevations are obtained by first drawing side views of the objects, as shown in Fig. 249. The connection is fully shown in Figs. 254 A and B.
181. To project a quadrilateral prism $6 \frac{1^{\prime \prime}}{4} \times 3 \frac{1}{2}{ }^{\prime \prime} \times 3 \frac{1}{2}{ }^{\prime \prime}$ resting on one corner on H.P., and its faces forming equal angles with it, with its longer edges inclined at an angle of $60^{\circ}$ to H.P., and parallel to V.P. Its nearest edge to be $1_{\frac{1}{4}}{ }^{\prime \prime}$ from V.P. Scale $\frac{1}{8}$ full size. Fig. 256 A.

Draw XY, and $I_{4}{ }^{\prime \prime}$ below it draw the line $m n$. In any convenient position draw the line $c a$ perpendicular to $m n$, and from $c$ draw $c b 3 \frac{1}{2}^{\prime \prime}$ long, at an angle of $45^{\circ}$. From $b$ as centre, and with radius $b c$, draw an arc cutting $c a$ in $a$. Join $b a$. Also draw $b d$ perpendicular to $c a$. This represents one-half of the actual shape of the base of the prism.


At any point $e^{\prime}$ on XY draw the line $e^{\prime} f^{\prime} 6 \frac{1}{4}{ }^{\prime \prime}$ long at an angle of $60^{\circ}$ to XY , and the line $e^{\prime} h^{\prime}$ perpendicular to it. From $e^{\prime}$, along $e^{\prime} h^{\prime}$, set off the distances $e^{\prime}, g^{\prime}, h^{\prime}$, equal to those of $a, d, c$. At each of these points draw lines parallel to $e^{\prime} f^{\prime}$, and equal to it. Join the tops of these lines. This completes the elevation. Draw lines from $b$ and $a$ parallel to $m n$. Every point in the plan must come on one of these three lines. Drop a line from $f^{\prime}$ at right angles to XY till it meets the horizontal line from $b$; this gives the point $f$. Every other point in the plan is found in the same manner.
182. To project the same prism, with its longer edges still inclined at an angle of $60^{\circ}$ with H.P., and its faces making equal angles with it; but instead of being parallel to V.P., as in Fig. 256 A, let them be in vertical planes, inclined at an angle of $45^{\circ}$ with V.P. The nearest corner of prism to be $1_{4}^{\frac{1}{4}}$ from V.P. Scale $\frac{1}{8}$ full size. Fig. 256 B.
Note.-We can always imagine any line to be contained in a vertical plane, whether the object contains one or not. In Fig. 254 B the line $k^{\prime} o^{\prime}$ is contained in the vertical plane $k^{\prime} o^{\prime} l^{\prime} g^{\prime}$; but in this instance $s^{\prime} w^{\prime}$ (Fig. 256 B) is not contained in one, as the solid does not contain a vertical plane.

At any point $o \mathrm{I}_{4}^{\frac{1}{\prime \prime}^{\prime \prime}}$ below XY, draw oq at an angle of $45^{\circ}$ with XY. Make $o q$ equal in length to $m m$ (Fig. 256 A ). The plan Fig. 256 B is precisely similar in shape to the plan Fig. 256A, but turned to make an angle of $45^{\circ}$ with XY ; so if we take the line $q o$ to represent the line $m n$, we can complete the plan from Fig. 256 A.

Every point in the elevation is found in the same way : erect a perpendicular upon point $r$ till it meets a horizontal line drawn from $f^{\prime}$ (Fig. 256A) ; this gives the point $r^{\prime}$, and so on till the elevation is completed.
183. To project a regular hexagonal prism $10^{\prime \prime}$ long, and with sides
 $3 \frac{1}{4}$ " wide, standing on its base on H.P., with one face parallel to V.P. and $2 \frac{1}{2}^{\prime \prime}$ from it. Scale $\frac{1}{12}$ full size. Fig. 257 A .
Draw the line XY, and $2 \frac{1}{2}$ " below it draw the line $a b$. Complete the hexagon. Above XY draw perpendiculars io" long immediately above the points $c, e, f, d$. Join the tops of these perpendiculars, which completes the elevation.
184. To project the same prism lying on one face on the H.P., with its longer edges parallel to the V.P.; its nearest edge to be $1_{4}^{3 \prime \prime}$ from V.P. Scale $\frac{1}{12}$ full size. Fig. 257 B.
Draw the line $l m, 10^{\prime \prime}$ long $1 \frac{3 \prime \prime}{4}$ below XY and parallel to it. Draw the lines $l s$ and $m p$ perpendicular to $l m$. Set off the
distances $m, n, o, p$ on $m p$ equal to the distances $c^{\prime}, e^{\prime}, f^{\prime}, d$ (Fig. 257 A). Draw lines from $n, o$, and $p$, parallel to $l m$, tlll they meet the line $l s$. This completes the plan.

Draw perpendiculars above XY immediately above the points $s, p$, and set off the distances $u^{\prime}, s^{\prime}, r^{\prime}$ equal to the distances K, H, G (Fig. 257 A ). Draw lines from the points $s^{\prime}$ and $r^{\prime}$ parallel to XY till they meet the perpendicular $p^{\prime} o^{\prime}$. This completes the elevation.
185. To project a regular hexagonal prism $7 \frac{1}{2}{ }^{\prime \prime}$ long, and with sides $2 \frac{1}{2}^{\prime \prime}$ wide, standing on its base on the H.P., with one face inclined to the V.P. at an angle of $45^{\circ}$. Its nearest edge to be $1 \frac{3^{\prime \prime}}{4}$ from V.P. Scale $\frac{1}{9}$ full size. Fig. 258 A.
Draw the line XY, and $\mathrm{I} \frac{3^{\prime \prime}}{4}$ below it mark the position of point $a$.

From $a$ draw the line $a b, \mathrm{I}_{4}^{3^{\prime \prime}}$ long, at an angle of $45^{\circ}$ with XY. Complete the hexagon. Above XY draw perpendiculars $7 \frac{1}{2}^{\prime \prime}$ long immediately above the points $b, c, d, e$, and
 join the tops of these lines.

This completes the elevation.
186. To project the same prism, lying with one face on H.P.; its longer edges to be inclined to the V.P. at an angle of $30^{\circ}$; its nearest corner to be $1_{\frac{1}{4}}{ }^{\prime \prime}$ from V.P. Scale $\frac{1}{9}$ full size. Fig. 258 B.
Fix the point $f \mathrm{I}_{\frac{1}{4}}{ }^{\prime \prime}$ below XY. Draw the line $f g$, $7 \frac{1^{\prime \prime}}{}$ long, at an angle of $30^{\circ}$ with XY ; and from $f$ and $g$ draw lines perpen. dicular to $f g$. From $g$, along the line $g k$, set off the distances $g, m, l, k$, equal to the distances $c, i, p, o$ (Fig. 258A). From the points, $m, l$, and $k$, draw lines parallel to $f g$ till they meet the line $f h$. This completes the plan.

The heights $K, H, G$, which give the horizontal lines in the elevation, are obtained from the distances $b, i, d$ in the plan (Fig. 258 A ). Having obtained these heights, draw the lines $h^{\prime} k^{\prime}$ and $n^{\prime} m^{\prime}$. Carry up perpendicular lines from the points in the plan till they meet these lines, which give the corresponding points in the elevation. Join them as shown.
187. To project a regular hexagonal prism $12 \frac{1_{2}^{\prime \prime}}{}$ long, with sides $4^{\prime \prime}$ wide, resting on one of its smaller edges on the H.P.; its longer edges to be inclined at an angle of $60^{\circ}$ to the H.P., and parallel to the V.P. Its nearest edge to be $2^{\prime \prime}$ from V.P. Scale $\frac{1}{15}$ full size. Fig. 259 A.

In any convenient position draw the hexagon AEDF with $4^{\prime \prime}$ sides, with lines joining the opposite angles as shown. Draw
 XY, and at any point $a^{\prime}$ draw the line $a^{\prime} c^{\prime}, 12 \frac{1}{2}{ }^{\prime \prime}$ long at an angle of 60 with XY. From $a^{\prime}$ and $c^{\prime}$ draw the lines $a^{\prime} b^{\prime}$ and $c^{\prime} d^{\prime}$ perpendicular to $a^{\prime} c^{\prime}$. Set off the distances $a^{\prime}, f^{\prime}, b^{\prime}$ along $a^{\prime} b^{\prime}$, equal to the distances $\mathrm{E}, \mathrm{B}, \mathrm{F}$ of hexagon. From these points draw lines parallel to $a^{\prime} c^{\prime}$ till they meet the line $c^{\prime} d^{\prime}$. This completes the elevation.
Draw a line $2^{\prime \prime}$ below XY, and parallel to it. From L on this line let fall a perpendicular, and on it set off the distances $\mathrm{L}, \mathrm{K}, \mathrm{H}, \mathrm{G}$ equal to the distances $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ of hexagon. From each of these points draw lines parallel to XY , and let fall lines from the various points in the elevation till they meet these lines. This gives the corresponding points in the plan. Join them as shown.
188. To project the same prism standing on one of its shorter edges on the H.P., at an angle of $30^{\circ}$ with V.P., with its longer edges inclined at an angle of $60^{\circ}$ with the H.P., and in vertical planes inclined at an angle of $60^{\circ}$ to V.P.; its nearest corner to be $3^{\prime \prime}$ from V.P. Scale $\frac{1}{1 \overline{5}}$ full size. Fig. 259 B.
Fix the position of point $m 3^{\prime \prime}$ below XY, and draw the line $m p$ at an angle of $30^{\circ}$ with XY. This represents the line KH in Fig. 259 A. The plan in Fig. 259 B is precisely the same as that shown in Fig. 259 A, turned to a different angle. Complete the plan against the line $m p$, from Fig. 259 A . As every point in the elevation is found in the same way, it is only necessary to describe one point. Erect a line on point $r$ at right angles to XY till it meets a horizontal line drawn from point $e^{\prime}$. This gives the point $r^{\prime}$. Find the other points in the same way, and join them, as shown.
189. To project a regular hexagonal prism, $7_{\frac{1}{4}}{ }^{\prime \prime}$ long, with faces $2 \frac{1}{2}{ }^{\prime \prime}$ wide, resting on one corner on the H.P. ; its longer edges to be inclined at an angle of $45^{\circ}$ with the H.P.; one face to be parallel to the V.P. and $2 \frac{1}{2}{ }^{\prime \prime}$ from it. Scale $\frac{1}{9}$ full size. Fig. 260 A .
Construct a hexagon with $2 \frac{1}{2}$ " sides, and join the opposite

angles by lines at right angles to each other, as shown. Draw the line XY, and at any point $a^{\prime}$ draw the line $a^{\prime} b^{\prime}, 7 \frac{1}{4}^{\prime \prime}$ long, at an angle of $45^{\circ}$ with XY. From the points $a^{\prime}$ and $b^{\prime}$ draw the
lines $a^{\prime} c^{\prime}$ and $b^{\prime} d^{\prime}$ perpendicular to $a^{\prime} b^{\prime}$. Set off the distances $b^{\prime}, e^{\prime}, f^{\prime}, d$ on $b^{\prime} d^{\prime}$ equal to A, B, C, D of hexagon, and draw lines from these points parallel to $b^{\prime} a^{\prime}$ till they meet the line $a^{\prime} c$. This completes the elevation.

Draw the line $m k 2 \frac{1}{2}$ " below XY. From H on $m k$ produced draw the line HL perpendicular to $m k$, and set off the distances H, D, L equal to the distances E, B, F of hexagon. Draw lines from these points parallel to XY. All the points of the plan must come on these three lines, and are determined by dropping perpendiculars from the corresponding points in the elevation.
190. To project the same solid, with its longer edges still inclined at an angle of $45^{\circ}$ to the H.P.; but instead of being parallel to the V.P., as in the last problem, let them be in vertical planes, inclined at an angle of $30^{\circ}$ with the V.P.; its nearest corner to the V.P. to be $1 \frac{1}{4}{ }^{\prime \prime}$ from it. Scale $\frac{1}{9}$ full size. Fig. 260 B.
Fix the position of point $n \mathrm{I}_{4}^{1 \prime \prime}$ below XY, and draw the line no at an angle of $30^{\circ}$ with it. The plan in this problem is precisely similar to the plan in the last problem, but turned round at an angle of $30^{\circ}$ with XY, and the line no corresponds with the line km (Fig. 260 A). Complete the plan as shown.

Every point in the elevation is found in the same manner. For example, erect a perpendicular on point $o$ till it meets a horizontal line drawn from point $e^{\prime}$; this will give the point $o^{\prime}$. Proceed in the same way with all the other points, and join them.

## The Regular Solids.

There are five regular solids, and they are named after the number of faces they each possess; viz. the Tetrahedron has four faces, the Hexahedron six faces, the Octahedron eight faces, the Dodecahedron twelve faces, and the Icosahedron twenty faces.

They possess the following properties, viz. :
(1) The faces of each solid are equal, and similar in shape, and its edges are of equal length. (2) All their faces are regular polygons. (3) All the angles formed by the contiguous faces of each solid are equal. (4) Each can be inscribed in a sphere, so that all its angular points lie on the surface of the sphere.

The student is advised to make these drawings to a scale three times that of the diagrams.
191. To project a tetrahedron with edges $9^{\prime \prime}$ long, with one face resting on the H.P.; one of its edges to be at an angle of $16^{\circ}$ with V.P., and its nearest angular point $33^{\frac{3}{4}}$ from V.P. Scale $\frac{1}{12}$ full size. Fig. 261 A.
All the faces of this solid are equilateral triangles.
Draw XY, and fix the position of point $a 3 \frac{3^{\prime \prime}}{4}$ below it. From $a$ draw the line $a b$ $9^{\prime \prime}$ long, at an angle of $16^{\circ}$ with XY. On the line $a b$ construct an equilateral triangle $a b c$. Bisect each of the angles at $a, b$, and $c$ by lines meeting at $d$. This completes the plan.

To find the altitude of the elevation, produce the line $b d$ to $e$, and at $d$ draw the line $d f$ perpendicular to $e b$.


A Fig. 26r. With $b$ as centre, and radius $b c$, draw an arc cutting $d f$ in $f$.

To draw the elevation, erect a perpendicular $d^{\prime} g^{\prime}$ directly above $d$, and make $d^{\prime} g^{\prime}$ equal to $d f$. Carry up the points $a, c, b$ till they meet XY in $a^{\prime}, c^{\prime}$, and $b^{\prime}$. Join the point $d^{\prime}$ to $a^{\prime}, c$, and $b^{\prime}$.
192. To project the same solid tilted on to one of its angular points with its base inclined at an angle of $20^{\circ}$ with H.P. Scale $\frac{1}{12}$ full size. Fig. 26I B.
Fix the point $k^{\prime}$ on XY, and draw the line $k^{\prime} h^{\prime}$ at an angle of $20^{\circ}$ with it, and equal in length to the line $b^{\prime} a^{\prime}$ (Fig. 26I A). As this elevation is precisely similar to the last, complete it from that figure on the line $h^{\prime} k^{\prime}$.

Draw lines from the points $a, b, d$, and $c$ (Fig. 261 A) parallel to XY. All the points of the plan must come on these four lines, and are found by dropping lines at right angles to XY from the corresponding points in the elevation.
193. To project a hexahedron or cube with edges $3^{3{ }^{\prime \prime}}$ long, resting on one edge on the H.P., and its base making an angle of $22^{\circ}$ with it; the side nearest the V.P. to be parallel to, and $1 \frac{3}{4}^{\prime \prime}$ from it. Scale $\frac{1}{6}$ full size. Fig. 262 A.

All the faces of this solid are squares.
Draw the line XY, and at any point $a^{\prime}$ draw the line $a^{\prime} b^{\prime}$ at an angle of $22^{\circ}$. Complete the square $a^{\prime} b^{\prime} c^{\prime} d^{\prime}$.


Below XY draw the line ef $\mathrm{I} \frac{3^{\prime \prime}}{}{ }^{\prime \prime}$ from it. Draw the lines $a^{\prime} a, b^{\prime} b$; $c^{\prime} c$ and $d^{\prime} d$ at right angles to XY. Make $e b$ and $f d$ each equal to $a^{\prime} b^{\prime}$, and join $b d$; $c c^{\prime}$ cuts ef in $g$.
194. To project the same solid with its edge still resting on the H.P. but inclined to the V.P. at an angle of $60^{\circ}$, the base still forming the same angle with the H.P., viz. $2^{\circ}$; its nearest corner to be $1^{\prime \prime}$ from V.P. Scale $\frac{1}{6}$ full size. Fig. 262 B.
Fix the position of point $h \mathrm{I}^{\prime \prime}$ below XY , and draw the line $h k$ equal to $f e$ (Fig. 262 A ) at an angle of $30^{\circ}$ with XY. Complete this plan from Fig. 262 A, to which it is precisely similar in shape.

Draw lines through the points $b^{\prime}, c^{\prime}$, and $d^{\prime}$ (Fig. 262 A ) parallel to XY. Draw perpendiculars from the points in the plan till they meet these lines, the intersections give the corresponding points in the elevation. Join these points as shown.
195. To project an octahedron, with edges $4^{\prime \prime}$ long, poised on one of its angular points on the H.P.; its axis to be perpendicular to H.P., and its edge nearest to the V.P. to be parallel to, and $1 \frac{3}{4}{ }^{\prime \prime}$ from it. Scale $\frac{1}{6}$ full size. Fig. 263 A.

All the faces of this solid are equilateral triangles.
Draw the line XY, and $\mathrm{I}_{4}^{\frac{3 \prime}{\prime \prime}}$ below it draw the line $a b 4^{\prime \prime}$ long. Complete the square abdc, and draw its diagonals. This will be the plan of the solid.

As every angular point is equidistant from the centre of a sphere circumscribing the solid, the point $e$ must be the centre of the plan of the sphere and ad its dianneter.

Draw the projector $e e^{\prime}$ from the point $e$. Make $e^{\prime} f^{\prime}$ equal in length to ad . This will be the axis of the solid.


A
Fig. 263.
B

Bisect $e^{\prime} f^{\prime}$ by the line $c^{\prime} d^{\prime}$ parallel to XY , and erect perpendiculars from the points $c$ and $d$ till they meet this line in the points $c^{\prime}$ and $d^{\prime}$. Join $c^{\prime} e^{\prime}$ and $c^{\prime} f^{\prime}, d^{\prime} e^{\prime}$ and $d^{\prime} f^{\prime}$, to complete the elevation.
196. To project the same solid with one face resting on the H.P.; the edge nearest the V.P. to be parallel to, and $13^{\prime \prime \prime}$ from it. Scale $\frac{1}{6}$ full size. Fig. 263 B.
Fix the positions of the points $g^{\prime}$ and $h^{\prime}$ on XY the same distance apart as the points $c^{\prime}$ and $f^{\prime}$ in the preceding problem. On this line complete the elevation from Fig. 263 A.

Draw lines from the points $b, e$, and $d$ parallel to XY. All the points of the plan must come on these three lines, and are found by dropping perpendiculars from their corresponding points in the elevation.

The Dodecahedron and Icosahedron can hardly be described as 'simple solids.' Their projection will be found in Spanton's Complete Geometrical Course (Macmillan), pp. 229 sqq.

## Octagonal Pyramids.

197. To project a regular octagonal pyramid, $8^{\prime \prime}$ high, with each side of its base $2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ wide, standing on its base on the H.P., with an edge of its base parallel to the V.P. and $2 \frac{1}{2}$ " from it. Scale $1_{12}^{\frac{1}{2}}$ full size. Fig. 264 A.

Draw XY, and $2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ below it draw $a b 2 \frac{1}{2}^{\prime \prime}$ long. On $a b$ construct a regular octagon, and join the opposite angles


Carry up projectors from the points $d, e, f, g$ perpendicular to XY. Fix the point $i^{\prime}$ immediately above $c$, and $8^{\prime \prime}$ above XY. Join the point $c^{\prime}$ to $d^{\prime \prime}, e^{\prime}, f^{\prime}$, and $g^{\prime}$, which completes the pyramid.
198. To project the same solid lying with one face on the H.P., but with its axis in a plane parallel to the V.P. Scale $\frac{1}{1}$ I. full size. Fig. 264 B.

On XY mark off the distance $h^{\prime} k^{\prime}$ equal to $g^{\prime} C^{\prime}$ (Fig. 264 A ). Complete the construction of elevation from Fig. 264 A.

Let fall lines from the various points of the elevation, at right angles to XY, till they meet lines drawn from the corresponding points in the plan (Fig. 264 A), parallel to XY. The intersection of these lines give the required points, by joining which we obtain the plan.
199. To project the same solid resting on one of its shorter edges, with its base inclined at an angle of $30^{\circ}$ with H.P. ; its axis to be in a plane parallel to the V.P. Scale $\frac{1}{12}$ full size. Fig. 264 C.

Draw XY, and fix the position of point $o^{\prime}$. Draw $o^{\prime} p^{\prime}$ at an angle of $30^{\circ}$ with XY , and equal in length to $h^{\prime} n^{\prime}$ (Fig. $26 \not+\mathrm{B}$ ). Complete the elevation from Fig. 264 b.

In any convenient position draw the line DG perpendicular to XY , and set off upon it the distances D, E, F, G equal to $h^{\prime}, l^{\prime}$, $m^{\prime}, n^{\prime}$ (Fig. 264 B). Draw lines from these points parallel to XY. From the various points in the elevation let fall lines at right angles to XY till they meet these lines, which give the corresponding points in the plan.

## Cones.

Cones are projected in precisely the same way as polygonal pyramids. After finding the points in the base, instead of joining them by lines, as in the case of pyramids, a fair curve is drawn through them. Eight points of the base are found in the examples here given. Should more points be required, it is only necessary to select a pyramid having more sides than eight to construct the cone upon.
200. To project a cone $8^{\prime \prime}$ high, with base $6 \frac{1}{2}{ }^{\prime \prime}$ in diameter, resting on the H.P. Scale $\frac{1}{12}$ full size. Fig. 265 A.
Draw XY, and in any convenient position below it draw a circle $6 \frac{1_{2}^{\prime \prime}}{}$ in diameter, and a diameter $g h$ parallel to XY .


Fig. 265.
Carry up projectors from $g$ and $h$ till they meet XY; also from $k$, and produce $k e 8^{\prime \prime}$ above XY. Join $k^{\prime} g^{\prime}$ and $k^{\prime} h^{\prime}$ 。
201. To project the same solid resting on its edge, with its base inclined at an angle of $30^{\circ}$ with H.P.; its axis to be in a vertical plane parallel to the V.P. Scale $\frac{1}{12}$ full size. Fig. 265 B.

Inscribe the circle (Fig. 265 A ) in a square $a b c d$, the side nearest XY to be parallel to it. Draw diagonals, and through the centre draw diameters parallel to the sides of the square. Through the four points where the diagonals cut the circle draw lines parallel to the sides of the square.

Fix the point $m^{\prime}$ on XY, and draw the line $m^{\prime} r^{\prime}$ at an angle of $30^{\circ}$ with it. Set off the distances $m^{\prime} o^{\prime} p^{\prime} q^{\prime} r^{\prime}$ equal to the distances $h^{\prime}, n^{\prime}, e^{\prime}, l^{\prime}, g^{\prime}$ (Fig. 265 A). Complete the elevation from Fig. 265 A.

Let fall lines from the various points of the elevation at right angles to XY till they meet lines drawn from the corresponding points of the plan (Fig. 265 A). Draw a fair curve through the points forming the base, and lines from the vertex $s$ tangential to the base. This completes the plan.
202. To project the same solid, resting on its edge, with its base still inclined at an angle of $30^{\circ}$ with H.P., but with its axis in a vertical plane, inclined at an angle of $60^{\circ}$ with V.P. Scale $\frac{1}{12}$ full size. Fig. 265 C.

Draw the lines enclosing the base with the parallel lines intersecting each other where the diagonals cut the circle from the plan (Fig. 265 B). The line $x y$ that passes through the axis to be inclined at an angle of $60^{\circ}$ with XY. Complete the plan from Fig. 265 B.

Draw lines from the various points of the plan at right angles to XY till they meet lines drawn from the corresponding points of the elevation (Fig. 265 B). Complete the elevation as shown.

## Cylinders.

In the examples here given, only eight points of the circular base are projected, to save confusion of lines ; but any number of points can be found in the same manner.
203. To project a cylinder, $5 \frac{1^{\prime \prime}}{}$ in diameter and $8 \frac{1}{4}{ }^{\prime \prime}$ high, standing on its base on the H.P. Scale $\frac{1}{6}$ full size. Fig. 266 A .

Draw XY, and in any convenient position below it draw a sircle $5 \frac{1}{2}{ }^{\prime \prime}$ in diameter. Draw tangents to it at the points $a$ and $b$ perpendicular to XY , and produce them $8 \frac{1}{4}^{\prime \prime}$ above XY. Join the tops of these lines to complete the cylinder.
204. To project the same cylinder, lying on its side on the H.P., with its axis inclined at an angle of $45^{\circ}$ with the V.P. Scale $\frac{1}{6}$ full size. Fig. 266B.

Draw four diameters to the plan (Fig. 266A), by first drawing the line $a b$ parallel to XY,
 and then the other three diameters equidistant from it. This gives eight points in the circumference. Draw lines from these points parallel to XY , till they meet the line AE in the points A, B, C, D, E.

At any convenient point $c$ below XY (Fig. 266 B), draw the line of at an angle of $45^{\circ}$ with it. Draw the line $\mathrm{cm}, 8 \frac{11^{\prime \prime}}{}$ long, perpendicular to cf , and from $m$ draw $m g$ parallel to cf . From $m$, along $m g$, set off the distances $m, l, k, h, g$ equal to the distances A, B, C, D, E (Fig. 266 A), and from these points draw lines parallel to cm . This completes the plan.

Draw a line from $f$ perpendicular to XY , and produce it above the ground line. Set off the distances $\mathrm{E}^{\prime}, \mathrm{D}^{\prime}, \mathrm{C}^{\prime}, \mathrm{B}^{\prime}, \mathrm{A}^{\prime}$ on this line, equal to the distances E, D, C, B, A (Fig. 266 A), and draw lines from these points parallel to XY. Draw the projectors from the various points in the plan till they meet these lines, which give projections of the points required to complete the elevation.
205. To project the same solid, resting on its edge, with its base inclined at an angle of $30^{\circ}$ with the H.P., its axis being parallel to the V.P. Scale $\frac{1}{6}$ full size. Fig. 267 A.
Draw XY, and at any point $n^{\prime}$ upon it draw the line $n^{\prime} r^{\prime}$ at an angle of $30^{\circ}$ with the ground line. On the line $n^{\prime} r^{\prime}$ set off the distances $n^{\prime}, o^{\prime}, p^{\prime}, q^{\prime}, r^{\prime}$ equal to the distances $g, h, k, l, m$ (Fig. 266 B ), and from each of these points draw perpendiculars to $n^{\prime} r^{\prime}, 8 \frac{1^{\prime \prime}}{}$ long. Join $s^{\prime} w^{\prime}$. This completes the elevation.


From $r^{\prime}$ let fall a line at right angles to XY , and set off upon it from any convenient point A the points $\mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ equal to the distances $n^{\prime}, o^{\prime}, p^{\prime}, q^{\prime}, r^{\prime}$. From each of these points draw lines parallel to XY. From the various points in the elevation drop projectors till they meet these lines in the corresponding points, by connecting which we get the plan.
206. To project the same solid resting on its edge, with its base still inclined at an angle of $30^{\circ}$ with the H.P., but with its axis inclined at an angle of $60^{\circ}$ with the V.P. Scale $\frac{1}{6}$ full size. Fig. 267 B.
Draw the line FG equal to AE , inclined at an angle of $30^{\circ}$ with XY. Complete the plan from Fig. 267 A. Draw projectors from the plan till they meet lines drawn parallel to XY from the corresponding points in the elevation (Fig. 267 A). These give the necessary projections for completing the elevation.

## Spheres.

The plan and elevation of a sphere are circles; but if we divide the sphere into divisions by lines upon its surface, such as meridians of longitude and parallels of latitude, we shall be enabled to fix its position and inclination to the co-ordinate planes, and project it accordingly.

So as not to confuse the figure too much, we will restrict ourselves to eight meridians, with the equator, and two parallels of latitude. The junction of the meridians will of course give us the position of the poles, which will determine the axis.
207. To project a sphere $5 \frac{1}{2}{ }^{\prime \prime}$ in diameter, with meridians and parallels; its axis to be perpendicular to the H.P. Scale $\frac{1}{6}$ full size. Fig. 268 A .
Draw XY, and in any convenient position below it draw a circle $5 \frac{1}{2}$ " in diameter. Draw the diameters $a b$ parallel to the ground line, and $d t$ at right angles to it, and two other diameters equidistant from them.

Produce the line $d t$ above XY , and make $c^{\prime} p$ ' equal in
 length to the diameter $a b$. Bisect $c^{\prime} p^{\prime}$ in $d^{\prime}$. With $d^{\prime}$ as centre, and radius equal to $p a$, draw a circle. Draw $a^{\prime} b^{\prime}$ through $d^{\prime \prime}$ till it meets the circle in $a^{\prime}$ and $b^{\prime}$. Through $d^{\prime}$ draw the line $e^{\prime} f^{\prime}$ at an angle of $45^{\circ}$ with $a^{\prime} b^{\prime}$, till it meets the circle in $e^{\prime}$ and $f^{\prime}$. From $e^{\prime}$ and $f^{\prime}$ draw lines parallel to $a^{\prime} b^{\prime}$ till they meet the circle in $g^{\prime}$ and $h^{\prime}$. These lines represent parallels of latitude. Drop a perpendicular from $g^{\prime}$ till it meets $a \dot{b}$ in $g$. With $p$ as
centre, and radius $p g$, draw a circle. This is the plan of the parallel $c^{\prime} g^{\prime}$.


To avoid confusion, the pro jectors for half of one meridian only are shown ; but they are all found in the same manner.

Erect a perpendicular on point $k$ till it meets the equator in point $k^{\prime}$; also from point $l$ till it meets the parallels in points $l^{\prime \prime}$ and $l^{\prime}$. Draw a curve through the points $c^{\prime}, l^{\prime \prime}, k^{\prime}, l^{\prime}, p^{\prime}$, which gives the projection of the meridian.
208. To project the same sphere, with its axis inclined to the H.P. at an angle of $60^{\circ}$, but parallel to the V.P. Scale $\frac{1}{5}$ full size. Fig. 268 B.
Note.-The same letters are taken throughout these spherical problems to facilitate reference.

Draw a circle $5 \frac{1}{2}^{\prime \prime}$ in diameter, touching XY, and draw the line $c^{\prime} p^{\prime}$ at an angle of $60^{\circ}$ with it. Draw the line $a^{\prime} b^{\prime}$ at right angles to $c^{\prime} p^{\prime}$, and set off the distances of the parallels above and below $a^{\prime} b^{\prime}$ equal to their distances in the elevation (Fig. 268 A ). Draw the lines $e^{\prime} g$ and $h^{\prime} f^{\prime}$ parallel to $a^{\prime} b^{\prime}$. Complete the elevation from Fig. 268 A. Draw lines from all the points of intersection between the meridians and parallels of the plan (Fig. 268 A ) parallel to XY, and let fall perpendiculars from the corresponding points in the elevation till they meet these lines, which give the projections of the points of intersection. Draw the curves.
209. To project the same sphere, with its axis still inclined to the H.P., at an angle of $60^{\circ}$, but in a vertical plane inclined at an angle of $60^{\circ}$ with the V.P. Scale $\frac{1}{6}$ full size. Fig. 269.
Draw the line $c p$ at an angle of $60^{\circ}$ with XY , for the plan of the axis, and on this line complete the plan from Fig. 268 b. Perpendicular to XY draw the line XL, and set off the distances $\mathrm{X}, \mathrm{C}, \mathrm{L}, \mathrm{K}, \mathrm{C}, \mathrm{L}$ equal to the distances $\mathrm{Y}, \mathrm{C}, \mathrm{L}, \mathrm{K}, \mathrm{C}, \mathrm{L}$ (Fig. 268 B ). From each of these points draw lines parallel to XY till they meet projectors drawn from the corresponding points in the plan, which give the projections required.


Fig. ${ }^{269}$.

## EXERCISES.

1. The plan is shown of three bricks (Fig. 270), each $9^{\prime \prime} \times 4 \frac{1_{2}^{\prime \prime}}{} \times 3^{\prime \prime}$, one resting upon the other two. Draw an elevation upon the given line $x y$.

Scale (which need not be drawn) $2^{\prime \prime}$ to $\mathbf{I}^{\prime}$, or $\frac{1}{6}$ of full size. (April, '98.)


Fig. 270.


Fig. 271
2. The plan is given (Fig. 271) of a flight of three steps each $\frac{3^{\prime \prime}}{3}$ high, of which 3 is uppermost. Draw an elevation on the given $x y$.
3. Plan and elevation are given of a solid letter H (Fig. 272). Draw an elevation when the horizontal edge $a b$ makes an angle of 60 with the vertical plane of projection.
(June, '97.)


Fig. 272.


Fig. 273.
4. The diagram (Fig. 273) shows the plan of two square prisms, one resting upon the other. Draw their elevation upon the given $x y$. The lines $A B$ and CD are plans of square surfaces.
5. The plan is given of a cube (Fig. 274), having a cylindrical hole pierced through its centre. A vertical plane, represented by the line $l m$,


Fig. ${ }^{274}$.


Fig. 275. cuts off a portion of the solid. Draw an elevation on the line $x y$, supposing


Fig. ${ }^{276}$. the part of the solid in front of $l m$ to be removed. The part in section should be clearly indicated by lightly shading it. (April, '96.)
6. The plan is given (Fig. 275) of a right prism having equilateral triangles for its bases. These bases are vertical. Draw an elevation of the prism on the line $x y$. Show the form of the section of the prism made by the vertical plane $l \mathrm{~m}$. The part in section should be indicated by lightly shading it. (June, '98.)
7. Plan and elevation are given (Fig. 276) of a solid composed of a half-cylinder placed upon a prism. Draw a new elevation, when the horizontal edges of the prism make angles of $45^{\circ}$ with the vertical plane of projection.
(April, '96.)
8. An elevation is given (Fig. 277) of an archway with semi-circular head, in a wall $I^{\prime} 6^{\prime \prime}$ thick. Draw a second elevation upon a vertical plane which makes an angle of $45^{\circ}$ with the face of the wall. (Scale, which need not be drawn, $\frac{1_{2}^{\prime \prime}}{}$ to $\mathbf{I}^{\prime}$.)
(June, 'oo.)


Fig. 277.


Fig. 278.


Fig. 279
9. Plan and elevation are given (Fig. 278) of a rectangular block with a semi-cylindrical hollow in it. Draw a new elevation upon a vertical plane which makes an angle of $30^{\circ}$ with the horizontal edge $a b$. (April, '98.)
10. The diagram (Fig. 279) shows a side elevation of a square prism, pierced through its centre by a cylinder. Draw a front elevation of the solids.
(June, 'oo.)


Fig. 280.


Fig. 28r.
11. Plan and elevation are given (Fig. 280) of a cylinder through which a square opening has been cut. Draw a fresh plan and elevation of the solid, the plane of the circular base FG being inclined at $45^{\circ}$ to the vertical plane of projection. (June, '99.)
12. The "block" plan and end elevation are given (Fig. 28I) of a building having a square tower with pyramidal roof. A side elevation of the building is required.
13. The plan is given of a piece of cylindrical rod (Fig. 282) cut by a vertical plane shown at $l m$. Draw an elevation of the solid upon an $x y$ parallel to lm .
(April, '99.)


Fig. 282.


Fig. ${ }^{283}$
14. Plan and elevation are given of a sloping desk (Fig. 283). Draw an elevation upon a vertical plane parallel to the line $p q$. Show upon this elevation the outline of the section made by the vertical plane represented by $p q$.
15. The diagram (Fig. 284) shows the plan of two cubes, one resting upon the two others, with a sphere resting on the upper cube. Draw an elevation on the given $x y$.
(April, 'oo.)


Fig. 284.


Fig. 285.
16. Show in plan and elevation a shallow circular metal bath. Diameter at top $2^{\prime} 6^{\prime \prime}$, at bottom $2^{\prime}$, height $6^{\prime \prime}$. The thickness of the metal may be neglected. Scale (which need not be drawn) $\mathrm{I}^{\prime}$ to $\mathrm{I}^{\prime \prime}$. (April, '99.)
17. The diagram (Fig. 285) shows the elevation of a right cone having its vertex at V. Draw the plan.
(April, 'oo.)

## CHAPTER XIV.

## ORTHOGRAPHIC PROJECTION.

Preparatory to the study of sections of solids it is desirable to have a more thorough insight into the principles of Orthographic Projection, though its simpler applications need only be considered.

In Solid Geometry objects are projected by means of parallel projectors perpendicular to two co-ordinate planes. These planes may be considered as indefinite in extent. For instance, the H.P. might be extended beyond the v.p., and the v.p. below the H.P.
To understand this fully, let us take two pieces of cardboard about $12^{\prime \prime}$ square, and half-way across the middle of each cut a groove, as shown in Fig. 286. By fitting these two pieces


Fig. 286. together we obtain two planes intersecting each other at right angles, as shown in Fig. 287.
We have now four sets of co-ordinate planes, forming four "dihedral angles," identified by the letters A, B, C, D.
The angle formed by the upper surface of the H.P. with the front of the V.P. is called the "first dihedral angle," viz. A, fig. 287.

The angle formed by the upper surface of the H.P. with the back of the v.p. is called the "second dihedral angle," viz. B, fig. 287.

The angle formed by the under surface of the H.P. with the back of the v.P. is called the "third dihedral angle," viz. C, fig. 287.

The angle formed by the under surface of the H.P. with the front of the V.P. is called the " fourth dihedral angle," viz. D, fig. 287.

We will now take a piece of cardboard $4^{\prime \prime} \times 3^{\prime \prime}$, and place one of its shorter edges against the H.P. and a longer edge against the V.P. in the first


Fig. ${ }^{287}$. dihedral angle, with its surface perpendicular to each plane (Fig. 287). Let the corner A represent a point we wish to project on to each plane: the top edge $\mathrm{A} a^{\prime}$ represents its vertical projector, and the point $\alpha^{\prime}$ its vertical projection; the edge $\mathrm{A} \alpha$ represents its horizontal projector, and the point $a$ its horizontal projection, and so on, placing the cardboard in each of the dihedral angles.

## Lines.

To illustrate the projection of lines, we will restrict ourselves to the two co-ordinate planes
 of the first dihedral angle (Fig. 288).

Take the piece of cardboard and place it with one of its shorter edges on the H.P., with its surface parallel to the V.P. Let the top edge AB represent a line we wish to project. The edges A $a$ and $\mathrm{B} b$ will then represent the horizontal projectors, and the line $a b$ its horizontal projection. If we draw lines $\mathrm{A} a^{\prime}$ and $\mathrm{B} b^{\prime}$ perpendicular to the V.P. from the points $A$ and $B$, they represent the vertical projectors, and the line $a^{\prime} b^{\prime}$ its vertical projection.

We will now place the piece of cardboard touching both planes, with one of its shorter edges on the H.P., and its surface
perpendicular to both planes. Let the edge $\mathrm{C} c$ represent the line to be projected, $\mathrm{C} c^{\prime \prime}$ and $c c^{\prime}$ represent the vertical projectors, and the line $c^{\prime} c^{\prime \prime}$ its vertical projection. The point $c$ on the H.P. is called the "horizontal trace" of the line.

NoTE.-The point where a line, or a line produced, would meet either plane is called the "trace" of that line : if this point is on the H.P., it is called the "horizontal trace" (н.т.); and if it is on the V.P., the "vertical trace" (V.T.). The same thing applies to the projection of planes.

Now place the piece of cardboard with one of its longer edges on the H.P., and its surface perpendicular to both planes. Let the top edge $\mathrm{D} d^{\prime \prime}$ represent the line to be projected. The edges $\mathrm{D} d$ and $d^{\prime \prime} d^{\prime}$ represent the horizontal projectors, and the line $d d^{\prime}$ its horizontal projection. The point $d^{\prime \prime}$ is its vertical trace.

Fig. 289 represents the coordinate planes opened out into one flat surface. The projections below XY represent


Fig. 289. the plans of the lines, and those above XY the elevations.

We will now use the same piece of cardboard to illustrate the projections of lines inclined to one or both co-ordinate planes


Fig. 290.
(Fig. 290). In the first case we will incline it to both planes, with one of its shorter edges resting on the H.P. and parallel to the V.P.

Let the edge AB represent the line to be projected. $a \mathrm{~B}$ is its horizontal, and $a^{\prime} b^{\prime}$ its vertical projections.

Then incline it to the H.P., with one of its shorter edges still on the H.P., but perpendicular to the V.P.

Let $C D$ represent the line to be projected. The line $D c$ is the horizontal, and $c^{\prime} d^{\prime}$ its vertical projections.

Now incline it to the V.P., with its lower longer edge parallel to, but raised a little above the H.P.

Let EF represent the line to be projected. The line ef is its horizontal, and $f^{\prime} e^{\prime}$ its vertical projections.

Let us now draw a diagonal GF across the piece of cardboard and again hold it in the same position ; and let GF represent the line to be projected.

The line ef still represents its horizontal projection, but the line $f^{\prime} g^{\prime}$ is its projection on the vertical plane.

Fig. 29I shows the plans and elevations of these lines, with the co-ordinate planes opened out flat.

We have now projected a line in seven distinct positions, viz. :
Fig. 288.-AB parallel to H.P. and parallel to V.P.

C perpendicular to
D parallel to
Fig. 290. - AB inclined to CD
EF parallel to
GF inclined to


Fig. 291.

| 99 | 93 | 99 |
| :---: | :---: | :---: |
| 9 | perpendicular to | 99 |
| 9 | inclined to | 99 |
| 99 | parallel to | 93 |
| 99 | inclined to | 93 |
| $99^{\circ}$ | 99 | 3 |

The student should particularly notice the difference between AB and FG in Fig. 290. Although they are both inclined to both planes, $A B$ is in a rertical plane perpendicular to the V.P., while FG is in one inclined to the V.P.

We will now project these lines in the various positions to scale.
210. To project a line $A B 2 \frac{1^{\prime \prime}}{4}$ long, parallel to both the H.P. and V.P., its distances to be $3^{\prime \prime}$ from the H.P. and $1 \frac{1^{\prime \prime}}{}$ from the V.P. Scale $\frac{1}{6}$ full size. Fig. 292.
Draw XY, and $\mathrm{I} \frac{1_{2}^{\prime \prime}}{}$ below it draw the line $a b 2 \frac{1}{4}{ }^{\prime \prime}$ long. Draw the projectors $a a^{\prime}$ and $b b^{\prime}$ at right angles to XY , and $3^{\prime \prime}$ above it. Join $a^{\prime} b^{\prime}$.
211. To project a line CD $3 \frac{3^{\prime \prime}}{4}$ long to the same scale, parallel to the V.P. and $2 \frac{l^{\prime \prime}}{4}$ from it, but perpendicular to the H.P. Fig. 292.
Fix the position of point $c 2 \frac{1^{\prime \prime}}{4}$ below XY. Draw a line perpendicular to XY, and produce the same $3 \frac{3}{4}^{\prime \prime}$ above it. $c$ is the plan or H . trace, and $c^{\prime} d^{\prime}$ the elevation required.


Fig. 292.
212. To project a line EF $3^{\prime \prime}$ long to the same scale, parallel to the H.P. and $2 \frac{1}{4}{ }^{\prime \prime}$ above it, but perpendicular to the V.P. Fig. 292.

Below XY, and perpendicular to it, draw the line ef $f^{\prime \prime} 3^{\prime \prime}$ long. Draw the projector $f^{\prime} e^{\prime} 2 \frac{1}{4}{ }^{\prime \prime}$ long. $e f^{\prime}$ is the plan, and $e^{\prime}$ the elevation or V . trace.
213. To project a line GH $3^{\prime \prime}$ long to the same scale, parallel to the V.P. and $1 \frac{1}{2}{ }^{\prime \prime}$ from it, but inclined to the H.P. at an angle of $60^{\circ}$ 。 Fig. 292.
At any point $g^{\prime}$ on XY draw the line $g^{\prime} h^{\prime} 3^{\prime \prime}$ long, and inclined to the H.P. at an angle of $60^{\circ}$. Let fall the projectors $g^{\prime} g$ and $h^{\prime} h$ at right angles to XY. Set off the points $g$ and $h \mathrm{I}_{\frac{1}{2}}{ }^{\prime \prime}$ below XY , and join them.
214. To project a line KL $3^{\prime \prime}$ long to the same scale, inclined to the H.P. at an angle of $60^{\circ}$, but in a vertical plane perpendicular to the V.P. Fig. 292.
Draw the line $k^{\prime} l$ at right angles to XY till it meets horizontal lines drawn from $\hbar^{\prime}$ and $\hbar . k^{\prime} l^{\prime}$ is the elevation, and $l^{\prime} l$ the plan.
215. To project a line $\mathrm{MN} 3^{\prime \prime}$ long to the same scale, parallel to the H.P. and $1 \frac{1^{\prime \prime}}{}$ above it, but inclined to the V.P. at an angle of $45^{\circ}$. The end of the line nearer the V.P. to be $\frac{3^{\prime \prime}}{4}$ from it. Fig. 292.
Fix the point $n \frac{3^{\prime \prime}}{4}$ below XY, and draw $n m 3^{\prime \prime}$ long at an angle of $45^{\circ}$ with it. Carry up the projectors perpendicular to XY, and produce them $I_{\frac{1}{2}}^{\prime \prime}$ above it in the points $m^{\prime}$ and $n^{\prime}$. Join $m^{\prime} n^{\prime}$.
216. To project a line OP $3^{\prime \prime}$ long to the same scale, inclined to the H.P. at an angle of $30^{\circ}$, but in a vertical plane inclined to the V.P. at an angle of $60^{\circ}$; one end of the line to be on XY. Fig. 292.
From point $o^{\prime}$ on XY draw a line $o^{\prime} \mathrm{A} 3^{\prime \prime}$ long, and inclined to XY at an angle of $60^{\circ}$. From the same point $o^{\prime}$ draw the line $o^{\prime} \mathrm{B}$ at an angle of $30^{\circ}$ with XY. With $o^{\prime}$ as centre, and radius $o^{\prime} \mathrm{A}$, draw an arc till it meets $o^{\prime} \mathrm{B}$ in B . Draw the line BC perpendicular to XY . With $o^{\prime}$ as centre, and radius $o^{\prime} \mathrm{C}$, draw an arc till it meets $o^{\prime} \mathrm{A}$ in $p$. Draw the projector $p p^{\prime}$ till it meets a horizontal line drawn from B in $p^{\prime}$. Join $p^{\prime} o^{\prime}$. $o^{\prime} p$ is the plan, and $o^{\prime} p^{\prime}$ the elevation of the line required.

## Planes.

The lines in which planes intersect the co-ordinate planes are called traces: if on the H.P., the horizontal trace (H.T.) :


Fig. 293.
and on the v.p., the vertical trace (v.T.). The inclination of planes is determined by means of these traces.

We will take the same piece of cardboard that we have used for our previous illustrations and place it on the H.P. and parallel to the V.P., as A (Fig. 293). The line $a b$, where it intersects the H.P., will be its H.T.

If we place it parallel to the H.P. and perpendicular to the V.P., as B , the line $c^{\prime} d^{\prime \prime}$, where it intersects the V.P., will be its V.T.

By placing it perpendicular to each plane, as C, of will be its H.T. and $e g^{\prime}$ its V.T.

On opening these coordinate planes out flat these traces will appear as shown in Fig. 294.

We will now place the


Fig. 294. piece of cardboard perpendicular to the H.P. and inclined to the V.P., as D (Fig. 295) : $h k$ will then be the H.T., and $h l^{*}$ the V.T.


Fig. 295 .

Now incline it to the H.P. and make it perpendicular to the V.P., as E (Fig. 295) : mn will be its H.T. and $m o^{\prime}$ its V.T.

By inclining it to both planes with its shorter edges parallel to XY , as F (Fig. 295), $p q$ will be the H.T. and $r^{\prime} s^{\prime}$ the V.T.

For our next illustration we will take a $60^{\circ}$ set-square $G$, as a right angle will not fit closely to the two planes in this position ; tu will be the H.T. and $t w w^{\prime}$ the V.T.

If we now open the planes as before, these traces will be shown as in Fig. 296.


Fig. 296.
From these illustrations we can deduce the following facts-
A plane can have no trace on the plane it is parallel to (see A and B, Fig. 293.

If traces are not parallel to XY , they must intersect each other on that line (see D, E, and G, Fig. 295).

If the traces of a plane are in one straight line when the H.P. and V.P. are opened out so as to form one continuous surface, the angles the plane forms with each co-ordinate plane must be equal.

When a plane is perpendicular to either co-ordinate plane, its inclined trace will always give the amount of its inclination to the other co-ordinate plane ; e.g. hk (Fig. 296) forms with XY the angle $\phi$ or the inclination of D (Fig. 295) to the V.P., while $m o^{\prime}$ forms with XY the angle $\theta$ or the inclination of E to the H.P.

When a plane is inclined to both planes, but has its traces parallel to $X Y$, the sum of its inclinations, i.e. $\theta+\phi=90^{\circ}$; as F (Fig. 295).

The traces of planes inclined to one or both planes are not supposed to finish at XY; they are indefinite, and are generally produced a little beyond XY.
217. To find the traces of the following planes. Scale $\frac{1}{4}$ full size.

## Fig. 297.

A, $3^{\prime \prime} \times 2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ perpendicular to the H.P. and inclined to the V.P. at $60^{\circ}$. $\mathrm{B}, 3 \frac{3^{\prime \prime}}{4} \times \mathrm{I} \frac{3^{\prime \prime}}{4} \quad, \quad, \quad$ V.P. $\quad, \quad$ H.P., $45^{\circ}$. C, $4 \frac{1}{2}^{\prime \prime} \times 3^{\prime \prime}$ inclined $\quad, \quad$ H.P. at $60^{\circ}$ with shorter edges parallel to each plane.
Draw XY ; and at any convenient point $a$ draw $a b 3^{\prime \prime}$ long, and at an angle of $60^{\circ}$ with XY. From a draw $a c^{\prime} 2 \frac{1}{4}^{\prime \prime}$ long. Then $a b$ is the H.T., and $a c^{\prime}$ the V.T. of A.


Fig. 297.

From any convenient point $d$ draw $d f^{\prime} 33^{\prime \prime}$ long, and at an angle of $45^{\circ}$ with XY. From $d$ draw $d e ~ 1^{\frac{3}{4}}$. long perpendicular to $x y$. Then $d e$ is the H.T., and $d f^{\prime}$ the V.T. of B.

Take any point $g$ on XY, and draw $g h^{\prime} 4 \frac{1^{\prime \prime}}{}$ long at an angle of $60^{\circ}$ with it. From $h^{\prime}$ draw $h^{\prime} k$ perpendicular to XY; and from $g$ as centre, with radius $g k$, draw an arc till it meets a perpendicular from $g$ in $l$. From $g$ draw a perpendicular till it meets a horizontal line from $h^{\prime}$ in $m^{\prime}$. Draw $m^{\prime} n^{\prime}$ and $l o$, each $3^{\prime \prime}$ long, parallel to XY. Then $m^{\prime} n^{\prime}$ will be the V.T., and ol the H.T. of C.

We will now proceed with planes that are inclined to both planes of projection : they are called oblique planes. Let us take
a $60^{\circ}$ set square and place it so as to fit closely against both planes, as shown at A (Fig. 298). ca will be the H.T. and $c b^{\prime}$ the v.t.

The inclination of a plane to the co-ordinate plane containing its trace is the angle between two lines perpendicular to the trace, one in the co-ordinate plane and one in the plane itself.


Fig. 298.

The line $c a$ (Fig. 298) is the H.T. of the plane A, and $a b^{\prime}$ is a line in the plane A, and af a line in the H.P., both perpendicular to the H.T.; therefore $b^{\prime} a f$ is the angle A forms with the H.P.
218. To determine the traces of a plane inclined at an angle of $45^{\circ}$ to the H.P., and at an angle of $35^{\circ}$ to the V.P. Fig. 299.
Note.-In the Definitions (page io) a cone is described as being generated by the revolution of a right-angled triangle about one of its sides as an axis. The hypotenuse of this triangle is called a generatrix.

The problem is generally solved in the following manner: The generatrices of two cones forming the necessary angles to the two planes of projection are determined with their axes meeting at the same point on XY. The sides of these two cones should be tangential to a sphere, the centre of which is the
point on XY in which their axes meet. The plane required is tangential to the bases of these two cones.

Draw XY. Select any point $c$ for the point in which the axes of the cones meet, and draw a line through it at right angles to XY. At any point $d$ on XY draw a line at an angle of $60^{\circ}$ with it till it meets the perpendicular on $c$ in $b^{\prime}$. With $c$ as centre, and radius $c d$, draw the semicircle def. Join $f b^{\prime}$. Then def is the plan, and $f b^{\prime} d$ the elevation of a semi-cone.


Fig. 299.
From $c$ draw the line $c g^{\prime}$ perpendicular to $d b^{\prime}$. With $c$ as centre, and radius $c g^{\prime}$, draw a circle. This will represent the plan and elevation of a quarter of the enveloped sphere.

Draw the line $a h$, at an angle of $45^{\circ}$, tangential to the plan of the sphere, cutting $b^{\prime} c$ produced in $a$. With $c$ as centre, and radius ch, draw the semicircle $h k^{\prime} l$. Join al. Then hal will be the plan, and $l k^{\prime} h$ the elevation of another semi-cone.

From $a$ draw the line am tangential to the semicircle fed; i.e. the base of the horizontal semi-cone.

From $b^{\prime}$ draw the line $b^{\prime} m$ tangential to the semicircle $l k^{\prime} h$; i.e. the base of the vertical semi-cone.

Then $a m$ is the H.T. and $b^{\prime} m$ the v.t. required.

Fig. 300 is a perspective view showing this construction. The horizontal semi-cone is dotted in each instance.

There is another method of finding the traces for an oblique plane, viz. by first finding the projections of a line perpendicular


Fig. 300.
to the plane required, and then drawing the traces at right angles to these projections. This will be more easily understood by referring to the set-square B (Fig. 298).

Let $\mathrm{O} \phi$ represent a line at right angles to $o n^{\prime}$, and perpendicular to the plane $B$. Then op is the horizontal projection, and $o^{\prime} p$ the vertical projection of this line ( $\mathrm{O} p$ ) ; and the H.T. mo, and the v.T. $m n^{\prime}$, are at right angles to these two projections.
(For Exercises see p. 226.)

## CHAPTER XV.

## SECTIONS OF SOLIDS, CONSTRUCTION OF SECTIONAL AREAS.

A section is defined as the intersection of a solid by a plane. This plane is called the cutting plane, and in the following problems it is given inclined at different angles to both the co-ordinate planes. The surface of the solids cut through are projected, and the true shapes of the sectional area are " constructed."
219. To project a cube of $\frac{1_{2}^{\prime \prime}}{2}$ edge, standing on the H.P., and inclined at an angle of 30 to the V.P., intersected by a cutting plane inclined to the H.P. at an angle of $45^{\circ}$, and perpendicular to the V.P.; the plane to intersect both the horizontal faces of the cube. Fig. 301.
Draw the plan $a b c d$ of the cube, and carry up projectors from the points, $\frac{3^{\prime \prime}}{4}$ above XY , and join them for the elevation.

Find the traces of the cutting plane (Prob. 218 ).

Where the v.T. cuts the elevation in the points $e^{\prime}$ and $f^{\prime}$, drop projectors which will intersect the plan in the lines eh and $f g$. afgche is the plan of


Fig. 301. the cut surface of the cube, and $e^{\prime} a^{\prime} c^{\prime} f^{\prime}$ the elevation.

The sectional surface can be "constructed" by rotating the projecting surface of the section on either the H.T. or v.t.

To rotate the plan of the section on the H.T. Draw lines at right angles to H.T. from the points of the plan, and make the lengths of these lines from the H.T. equal to the distances of the corresponding points on the v.r. from XY; e.g. to obtain the point C , set off $k \mathrm{C}$ equal to $e^{\prime} c^{\prime}$, and so on with each of the other points. Join them, as shown, to complete the construction of the sectional surface.

To rotate the sectional surface on the V.T. Draw the perpendiculars from the points $e^{\prime}, a^{\prime}, c^{\prime}, f^{\prime}$, and make the lengths of these lines equal to the distances of the corresponding points below XY; e.g. make $f^{\prime} \mathrm{G}^{\prime}$ equal to $m g, f^{\prime} \mathrm{F}^{\prime}$ equal to $m f$, and so on with the other points. Join them as shown.
220. To project a quadrilateral prism $10 \frac{1}{2}$ " long, with a base $6^{\prime \prime}$ square, standing on its base on the H.P., with its longer edges parallel to the V.P., and one of its sides inclined to the V.P. at an angle of $60^{\circ}$. Intersect the prism with a plane parallel to XY , and inclined to


Fig. 302. the H.P. at an angle of $30^{\circ}$. Scale $\frac{1}{12}$ full size. Fig. 302.

Project the prism (Prob. 17I). Find the traces of the cutting plane (Prob. 218).

At any convenient point $e$ draw the line $\mathrm{E} e^{\prime}$ at right angles to XY cutting the H. and V. traces. With $e$ as centre, and radius $e \mathrm{E}$, draw an arc till it meets XY in $f$. Join $e^{\prime} f$. $\quad e f e^{\prime}$ is the angle the cutting plane forms with the H.P.

Draw lines from each point in the plan perpendicular to $e \mathrm{E}$, and meeting it in the points $\mathrm{C}, \mathrm{B}, \mathrm{D}, \mathrm{A}$. With $e$ as centre, and each of these points as radii, draw arcs; and where they meet XY draw perpendiculars till they meet $e^{\prime} f$ in the points $\mathrm{C}^{\prime}, \mathrm{B}^{\prime}, \mathrm{D}^{\prime}, \mathrm{A}^{\prime}$.

From these points draw horizontal lines till they meet projectors drawn from the corresponding points in the plan in the points $c^{\prime}, b^{\prime}, d^{\prime} a^{\prime}$. Join these points for the elevation of the section.

To construct the sectional area. Draw perpendiculars to $e^{\prime} f$ at the points $\mathrm{C}^{\prime}, \mathrm{B}^{\prime}, \mathrm{D}^{\prime}, \mathrm{A}^{\prime}$, and make their lengths equal to the distances of the corresponding points in the plan from the line $e \mathrm{E}$; e.g. make $\mathrm{C}^{\prime} \mathrm{C}^{\prime \prime}$ equal to $\mathrm{C} c, \mathrm{~B}^{\prime} \mathrm{B}^{\prime \prime}$ equal to $\mathrm{B} b$, and so on with the other points, and join them.
221. Project a regular hexagonal prism $9 \frac{1_{2}^{\prime \prime}}{}$ long, with $3 \frac{1}{2}{ }^{\prime \prime}$ sides, standing on its base on the H.P., with one of its faces inclined to the V.P. at an angle of $58^{\circ}$. Intersect the prism by a plane inclined at an angle of $55^{\circ}$ with the H.P., and $46^{\circ}$ with the V.P., the plane to cut through the base of the prism. Construct on the H.P. the sectional area. Scale $\frac{1}{1: 2}$ full size. Fig. 303.
Project the prism (Prob. 185). Find traces of cutting plane.
Draw lines from the points of the plan parallel to the H.т. till they meet XY, then draw perpendiculars to XY, till they meet the v.T. in the points $\mathrm{C}^{\prime}, \mathrm{A}^{\prime}, \mathrm{D}^{\prime}, \mathrm{F}^{\prime}, \mathrm{E}^{\prime}$. From these points draw horizontal lines till they meet projectors drawn from the corresponding points of the plan in the points $c^{\prime}, d^{\prime}, e^{\prime}, f^{\prime}, a^{\prime}, b^{\prime}$, and join them as shown. This is the elevation of the section.

To construct the true shape of the section, we rotate the plan $a b c d e f$ on the H.t. From $e$ draw the line $e g$ parallel to the H.T., and equal in length to the height of $e$ 'above XY.

Draw lines from each of the points in the plan at right angles to the H.T.: the one


Fig. 303. drawn from $e$ will intersect the H.T. in $h$. With this point as centre and the radius $h g$, draw an arc till it meets the line from $e$ produced in E. Find all the other points in the same way as shown.

The student should observe that the line $h \mathrm{E}$ is the true length of the line he. A simpler method of obtaining the length of these lines is as follows.

Note.-The true length of a projected line is equal to the hypotenuse of a right-angled triangle, the base of which is one of its projections, and the altitude the perpendicular height of the other projection.

Let $k d$ represent the horizontal projection of a line, and $\mathrm{K} d^{\prime \prime}$ its perpendicular height. From $k$ set off on the H.t. $k n$ equal to K $d^{\prime}$. Then the distance between $n$ and $d$ will be the true length of the line $k d$. Set this distance off from $k$ on $d k$ produced. This will give the point $D$. The other points can be found in the same manner.
222. Project a regular hexagonal prism $10 \frac{1_{2}^{\prime \prime}}{}$ long, with $3 \frac{1}{2}{ }^{\prime \prime}$ sides, lying on one of its faces on the H.P., with its longer edges inclined to the V.P. at an angle of $17^{\circ}$. Intersect the prism with a plane inclined at an angle of $50^{\circ}$ to the H.P. and $56^{\circ}$ to


Fig. 304. the V.P.; the plane to intersect all its longer edges. Construct on the V.P. its sectional area. Scale $\frac{1}{12}$ full size. Fig. 304.

Project the prism (Prob. 186).

Find the traces of the cutting plane (Prob. 218).

Let BC be the side of the prism resting on the H.P. Join AD. Then BC, AD, and EF are horizontal lines, and $\mathrm{B}^{\prime} \mathrm{A}^{\prime}$ and $\mathrm{B}^{\prime} \mathrm{F}^{\prime}$ their heights above XY.

Note.-A horizontal line contained by a plane would be drawn parallel to the H.T. on plan, and parallel to XY in elevation.

As $b c$ is on the H.P., it must coincide with the H.T. Where the lines $\mathrm{F}^{\prime}$ and $\mathrm{A}^{\prime}$ produced meet the v.T., drop perpendiculars
till they meet XY, and then draw lines parallel to the H.t. Where these lines intersect the lines of the plan they will determine the points of the section. Join them as shown.

Carry up projectors from these points till they meet the corresponding lines in the elevation, and join them.

NOTE.-The projectors are omitted in several of these problems to save confusion, but the points in plan and elevation bear corresponding letters throughout, so they can be easily recognised.

To construct the sectional area on the V.P. Draw lines from each of the points in the elevation at right angles to the v.t. Take the distance of point $e$ below XY as $e \mathrm{E}^{\prime}$, and set it off on the v.T. from $g$ as $g h$. Set off $g \mathrm{E}^{\prime \prime}$ equal to $h e^{\prime}$, the hypotenuse of a right-angled triangle, as described in the preceding problem. Find the other points in the same way, and join them as shown.
223. Project a regular pyramid standing on its base on the H.P., with its sides inclined to the V.P. at an angle of $45^{\circ}$; the cutting plane to be inclined at an angle of $43^{\circ}$ to the H.P. and $70^{\circ}$ to the V.P. Construct the sectional area on the V.P. Fig. 305.
Project the pyramid (Fig. 248).
Find the traces of the cutting plane (Prob. 218).
Produce the diagonal eg till it meets the H.T. in E. Draw the projector $\mathrm{EE}^{\prime}$, and from $\mathrm{E}^{\prime}$ draw a line parallel to the v.T. Where this line meets the edges of the pyramid in $b^{\prime}$ and $d^{\prime \prime}$ will determine two points in the section. Drop projectors from these points till they meet the corresponding lines of the plan in the points $b$ and $d$.

The lines forming the section of this pyramid are really the intersection lines of two planes; the cutting plane being one, and each side of the pyramid the other plane. We know that the line of intersection between two planes must have its trace where the traces of the two planes intersect. Produce three sides of the base of the pyramid till they meet the H.T. in the points $k, m$, and $l$. These are the traces of the lines required. Draw a line from $k$ through $b$, and produce it till it meets the diagonal $f / h$ in $a$. Draw lines from $m$ and $l$ in the same manner till they meet the diagonals in $d$ and $c$. Join $d c$. This will complete the plan of the section.

To determine the elevation of the points $a$ and $c$. With centre $o$, and radii $o a$ and $o c$, draw arcs till they meet the diagonal $e g$ in the points $n$ and $p$. Draw projectors to these points till they meet the edges of the pyramid in the points $n^{\prime}$


Fig. 305.
and $p^{\prime}$. Draw horizontal lines from these points till they meet the other edges of the pyramid in the points $c^{\prime}$ and $a^{\prime}$. Join the points as shown, to complete the elevation of the section.

Another method of obtaining the section of this pyramid is to assume a horizontal line $q^{\prime} r^{\prime}$ in any convenient position in the elevation, and drop a projector from $r^{\prime}$ till it meets the
diagonal $e g$ in $r$. Draw $r t$ parallel to the base $g h$. Produce $q^{\prime} r^{\prime}$ till it meets the V.T. in $s^{\prime}$. Draw a projector from $s^{\prime}$ till it meets XY in $s$. Draw a line from $s$ parallel to the H.T. till it meets the line $r t$ in $u$. Then $u$ is a point in the plan of the section, which can be completed from the traces $k, m$, and $l$, as previously described.

To construct the sectional area ABCD , proceed in the manner described in the preceding problem.
224. To project a section through a right vertical cone : the cutting plane to be perpendicular to the V.P.; to be inclined at an angle of $45^{\circ}$ to the axis of the cone, but not to intersect its base. Fig. 306. This section is an ellipse.

Let DE be the elevation of the section. Divide it into any number of equal parts, e.g. six. Draw the axis of the cone, and through the divisions on DE draw lines parallel to the base of the cone.

The plan of the section is determined by first finding $a$ succession of points in the curve, and then drawing a fair curve through them. We will take the points $b b$ as an example. With C as centre, and a radius equal to $\mathrm{G}^{\prime} \mathrm{H}$ (i.e. the radius of the cone at the level of $b^{\prime}$ ), draw an arc till it meets a projector drawn from $b^{\prime}$ in the points $b b$. Proceed in the same manner with the other points, and draw a fair curve through them.


Fig. 306.
To construct the sectional area. Draw the line $\mathrm{D}^{\prime} \mathrm{E}^{\prime}$ in any convenient position, parallel to DE , and draw lines from each of the divisions on DE at right angles to $\mathrm{D}^{\prime} \mathrm{E}^{\prime}$. Take the distance $\mathrm{G} b$ from plan, and set it off on each side of $\mathrm{G}^{\prime \prime}$ in the points $b^{\prime \prime} b^{\prime \prime}$. Find all the other points in the same manner, and draw a fair curve through them.
225. To project a section through a right vertical cone ; the cutting plane to be parallel to the side of the cone and perpendicular to the V.P. Fig. 307. This section is a parabola.
Let $D^{\prime} E^{\prime}$ be the elevation of the section. Divide it into any


Fig. 307. number of parts--it is better to have the divisions closer together towards the top. Draw horizontal lines through these divisions.

The plan is determined by finding a succession of points as in the preceding problem. We will take the points $b b$ as an example.

With $C$ as centre, and a radius equal to the semidiameter of the cone at the level of the division $b^{\prime}$, i.e. $\mathrm{G}^{\prime} \mathrm{H}$, draw an arc till it meets a projector from $b^{\prime}$ in the points $b b$. Find the other points in the same manner, and draw a fair curve through them.

To construct the sectional area. Draw the line $\mathrm{D}^{\prime \prime} \mathrm{E}^{\prime \prime}$ in any convenient position parallel to $\mathrm{D}^{\prime} \mathrm{E}^{\prime}$, and draw lines at right angles to it from the divisions on $\mathrm{D}^{\prime} \mathrm{E}^{\prime}$. Take the distance G $b$ from plan, and set it off on each side of $G^{\prime \prime}$ in the points $b^{\prime \prime} b^{\prime \prime}$. Proceed in the same manner with all the other points, and draw a fair curve through them.
226. To project a section through a right vertical cone; the cutting plane to be perpendicular to the H.P, and inclined at an angle of $50^{\circ}$ to the V.P. Fig. 308. This section is an hyperbola.
Let DE be the plan of the section. From C draw the line $\mathrm{C} d$ perpendicular to DE . With C as centre, and radius $\mathrm{C} d$, draw an arc cutting AB in $n$. Draw the projector $n n^{\prime}$. $n^{\prime}$ is the vertex of the section. Divide the height $g^{\prime} n^{\prime}$ into any number of divisions,-they should be made closer together near the vertex,-and draw horizontal lines through them till they meet the sides of the cone.

The elevation of the section is determined by first finding a succession of points, and then drawing a fair curve through them. We will take the points $b^{\prime} b^{\prime}$ as an example.

With the point $C$ on plan as centre, and a radius equal to $\mathrm{G}^{\prime} \mathrm{H}$ (the semi-diameter of the cone at the level of $b^{\prime}$ ), draw arcs


Fig. 308.
intersecting the line DE in the points $b b$. Draw projectors to these points till they meet the line drawn through $\mathrm{G}^{\prime} \mathrm{H}$ in the points $b^{\prime} b^{\prime}$. Find the other points in the same manner, and draw a fair curve through them.

If the cutting plane were perpendicular to both the co-ordinate planes, $g h$ would be the plan, and $g^{\prime} n^{\prime}$ the elevation of the section of the cone.

To construct the sectional area. Draw the line $n^{\prime \prime} g^{\prime \prime}$ in any convenient position parallel to the axis of the cone. Produce
the divisions on $n^{\prime} g^{\prime}$. Take the distance $d b$ from plan, and set it off on each side of $\mathrm{G}^{\prime \prime}$ in the points $b^{\prime \prime} b^{\prime \prime}$. Find all the other points in the same way, and draw a fair curve through them.

As the three preceding problems are conic sections, their sectional areas could be constructed by the methods described in Chap. XI. (Plane Geometry), but we must first determine the major and minor axes of the ellipse, and the directrices and foci of the parabola and hyperbola.

We will illustrate by a perspective view (Fig. 309) the principle of the relation between the directrix and focus of a


Fig. 309. parabola, and afterwards apply it to the ellipse and hyperbola.

ACB is a cone, and DEGF the cutting plane. H is a sphere touching the cutting plane, and inscribed in the upper portion of the cone. A line drawn from $c$, the centre of the sphere, perpendicular to the cutting plane, will meet it in $f$, which is the focus of the parabola.

The plane KLNM, containing the circle of contact between the sphere and the cone, intersects the cutting plane in the line $a g$, which is the directrix of the parabola.

A line joining the centre of the sphere with the circle of contact, as $c e$, is perpendicular to the side of the cone.

Compare this figure with Fig. 228 (Plane Geometry).
Let us now refer to Fig. 307. cfe is the inscribed sphere, $f$ is the point of contact with the cutting plane, ce is perpendicular to the side of the cone, and $e$ determines the level of the plane containing the circle of contact. A horizontal line drawn through $e$ till it meets the cutting plane produced in $d$ will determine the position of the directrix.

Draw a line from $f$ perpendicular to the cutting plane till it
meets the line $D^{\prime} E^{\prime}$ in $f^{\prime}$ : this is one of the foci of the ellipse. A line drawn from $d$ perpendicular to the cutting plane will determine the directrix ag.

The line $\mathrm{D}^{\prime} \mathrm{E}^{\prime}$ is the major axis of the ellipse, and if we bisect this line by another at right angles to it, and obtain the position of the points $K$ and $L$ in the same manner as we determined the points $b^{\prime \prime} b^{\prime \prime}$, KL will be the minor axis. We can obtain the other focus and directrix by setting off their distances on the opposite side of KL; or we could construct another sphere in the lower part of the cone, and obtain them as already described.

In Fig. 307, the same construction as previously described will determine the position of the directrix and focus; and as it bears corresponding letters, the student should have no difficulty in understanding it. Compare Fig. 307 with Fig. 309.

The same thing applies to Fig. 308.
227. To project the section of a right vertical cylinder ; the cutting plane to be inclined at angle of $36^{\circ}$ with the H.P. and $73^{\circ}$ with the V.P., but not intersecting the base. This section is an ellipse. Construct the sectional area on the H.P. Fig. 310.
Project the cylinder (Prob. 203).

Find the traces of the cutting plane (Prob. 218).

Assume a vertical plane passing through the axis of the cylinder, and perpendicular to the cutting plane. Draw $a b$ perpendicular to the H.t. Draw a projector to $a$ till it meets the v.T. in $a^{\prime}$. Draw a projector to $b$ till it meets XY in $b^{\prime}$. Join $a^{\prime} b^{\prime} . a b$ is the H.T. of this


Fig. 3ro.
v. plane, and $a^{\prime} b^{\prime}$ the line in which it intersects the cutting plane.

Draw a projector to $o$ till it meets $a^{\prime} b^{\prime}$ in $o^{\prime}$.

Divide the plan by diameters into eight equal parts, one of these diameters, $d h$, being in the H.T. of the V . plane. Projectors to $d h$ will give $d^{\prime}, h^{\prime}$, two of the points in the section. Produce the diagonal ie till it meets the H.T. of the cutting plane in $m$. Draw the projector $m m^{\prime}$, and draw a line from $m^{\prime}$ through $o^{\prime}$. This will give the corresponding points $i^{\prime}, e^{\prime}$, in the elevation.

Produce the diameter if till it meets XY in $n$. Draw a perpendicular to XY at $n$ till it meets the v.T. in $n^{\prime}$. Draw a line from $n^{\prime}$ through $o^{\prime}$. This will give the corresponding points $j^{\prime \prime}, f^{\prime}$ in the elevation. Find the points $c^{\prime}, g^{\prime}$ in the same manner.

Draw a fair curve through these points for the elevation of the section.

Any number of points in the curve could be found in the same manner by drawing additional diameters to the plan, but eight points are generally deemed sufficient.

To construct the sectional area. Find the points B, H, F, D, (Prob. 22I), and complete the ellipse (Prob. I 57, Plane Geometry).
228. To project the section of a sphere; the cutting plane to be inclined at an angle of $35^{\circ}$ with the H.P., and $74^{\circ}$ with the V.P. Fig. 3II.
Find the traces of the cutting plane (Prob. 218).
As a sphere is a continuous surface without any edges or angles, it will be necessary to assume certain fixed lines upon its surface in order to determine where the cutting plane will intersect it ; meridians and parallels are best suited for this purpose.

Project the sphere with meridians and parallels (Prob. 207). It will be better to arrange the meridians on plan so that one of them is parallel to the H.T.

Assume a V.P. perpendicular to the cutting plane and containing the axis of a sphere. Let $a b$ be the H.T. of this plane, and $a^{\prime} b^{\prime}$ the line in which it intersects the cutting plane. Where this line intersects the axis will determine $o^{\prime}$, and where it intersects the meridian $c^{\prime} d^{\prime}$ will give two points in the section.

Produce the meridian $j k$ on plan till it meets the H.T. in $n$. Draw the projector $u n^{\prime}$, and draw a line from $n^{\prime}$, through $o^{\prime}$. till it meets the meridian $j^{\prime} k^{\prime}$. These are two more points in the section.

Produce the meridian of till it meets XY in $l$. Draw a perpendicular at $l$ till it meets the v.T. in $l^{\prime \prime}$. Draw a line from $l^{\prime}$, through $o^{\prime}$, till it meets the meridian $f^{\prime} e^{\prime}$, giving two more points in the section. Obtain the points $h^{\prime}$ and $g^{\prime}$ in the same manner, and draw a fair curve through all the points found.


Fig. 311.
Drop projectors from each of these points till they meet the corresponding meridians on plan, and draw a fair curve through them.

These projections are ellipses. They could also be found by first projecting their conjugate diameters, and then completing them as in Problem 157.

The true shape of the section is of course a circle. To obtain its radius, bisect $c d$ in $s$, and draw a line $t u$ through $s$ parallel to the H.T.; st is the radius required ; tu is the major axis, and $c d$ the minor axis of the ellipse.

## EXERCISES.

1. Draw the plan of a right pyramid $2 \frac{1}{2}^{\prime \prime}$ high, base an equilateral triangle of $2^{\prime \prime}$ side. The pyramid stands on its base, and the upper part is cut off by a horizontal plane $1^{\prime \prime}$ above the base. Indicate the section clearly by light shading.
(April, '96.)
2. An elevation is given of a regular hexagonal prism with its bases horizontal (Fig 312). Draw its plan. Also show the true form of the section made by the inclined plane shown at lm .
(April, 'oo.)

3. The diagram (Fig. 313) shows the end elevation of a right prism ${ }^{\frac{3^{\prime \prime}}{4}}$ long with square base, and a horizontal plane $l m$ cutting the prism. Draw a plan of the portion below lm . (April, '98.)
4. The diagram shows (Fig. 314) the plan of a portion of a sphere. Draw an elevation upon the given line $x y$.
(June, '98.)



Fig. 315.
5. Plan and sectional elevation are given (Fig. 315) of a short length of moulding which has been cut across for "mitreing" by a vertical plane shown in plan at $l \mathrm{~m}$. Determine the true form of the section.
(June, '97.)
6. A right square pyramid, edge of base $\mathrm{I}_{\frac{1}{2}}{ }^{\prime \prime}$, height $2^{\prime \prime}$, is cut by a plane which contains one edge of the base, and is inclined at $45^{\circ}$ to the plane of the base. Draw the plan of the section, and if you can its true form.
(June, '99.)
7. A sphere of $\mathrm{I}^{\prime \prime}$ radius has a portion cut off by a horizontal plane $\frac{5}{5}^{\prime \prime}$ above its centre, and another portion by a vertical plane passing. $\frac{1}{2}^{\prime \prime}$ from the centre. Draw a plan of what remains of the sphere. The section must be clearly indicated by lightly shading it.
(June, '97.)
8. A right cone, height $2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$, radius of base $I^{\prime \prime}$, stands with its base on the horizontal plane. A sphere of $\mathbf{I}^{\prime \prime}$ radius rests on the horizontal plane and touches the cone. Draw a plan and elevation of the two solids, showing clearly your construction for finding the centre of the sphere. Show also the true form of the section of each solid by a horizontal plane $\frac{1_{2}^{\prime \prime}}{}$ above the centre of the sphere.
(April, 'o2.)
(For further Exercises see p. 228.)

## PART III.

## DESIGN.

## CHAPTER XVI.

T'He eye and the ear are both pleasantly affected by regularity of effect. The ear treats as music the even beats of the air, and in the same way the eye is a very accurate judge of evenness and regularity. It is strange how unevenness distresses the eye; for instance if the lines of this page were unequal distances apart, the effect would be very irritating.

But there is a further parallel between music and decoration. A discord has an effect on the ear which may be described as unsatisfying, and the "Resolution of the Discord" is called for to satisfy the demand for evenness, which has been temporarily violated.

In the same way, in decoration, the eye demands regularity, even uniformity of treatment; while at the same time it has a feeling that unevenness or the interruption of uniformity is pleasurable, if supplemented by regularity in the whole design. The two precisely similar spires of Cologne or Coutances are not altogether satisfying, because of their want of contrast; the eye is not challenged by contrast into attention to detail. This constitutes the special beauty of the twin, though dissimilar, spires of Lichfield or St. Lô, where the eye is piqued by the discord into the discovery of the prevailing harmony of design.

These principles should guide us in decorative design. Regularity must prevail in the main, with the unexpected to afford and challenge interest.

## Construction Lines on which Patterns are arranged.

To cover a wide space with regularity of form is an ancient device of man ; and in fact the old Greeks were better versed in its secrets than are we of this age. And yet, if we note their methods carefully, we may observe that geometrical balance was the secret of their success. To map out a surface with a geometrical network, and to fill it with balanced forms satisfying to the eye, seemed to them second nature. Slowly we are regaining the same instinct, and the beautiful decorations of modern artists are the product of diligent study of geometrical design.


Fig. 316.


Fig. ${ }^{18} 8$.


Fig. ${ }^{17} 7$.


Fig. 3 19.

A net is the simplest aid to mapping out a surface, and we have in Fig. 316 this groundwork of design, a square net like a tennis net; and in Fig. 317 a $45^{\circ}$ net, rectangular, and each line making equal angles of $45^{\circ}$ with the horizon. Fig. 318 is a $60^{\circ}$ net, covering the surface with a vertical diamond or lozenge: Fig. 319 a $30^{\circ}$ net, forming a horizontal diamond or lozenge. This mapping out of the whole surface by a network, or skeleton, must be the first step, as it is the geometric basis for the pattern, the unit of which will be the subject of discussion later.

The geometric framework must be prepared before the pattern which it is to carry. It will be noticed that in these nets certain lines are emphasized, showing how the simple net may be the basis of a more complicated network. A square net with


Fig. 320.


Fig. 32 r.


Fig. 322.


Fig. 323.


Fig. ${ }^{2}{ }^{2}$.


Fig. 325.


Fig. 326.


Fig. 327.
diagonals is shown in Fig. 320, and in Fig. 32I a $45^{\circ}$ net crossed by vertical and horizontal lines, which form octagons and other figures In both Figs. 320 and 321 a square net and a $45^{\circ}$ net are combined and certain lines are selected. In Fig. 322 , a $60^{\circ}$ net with horizontals forms a network of equilateral triangles or hexagons. Fig. 323 is a square lattice, and Fig. 324 a square plaid; Fig. 325 a lattice of $45^{\circ}$; Fig. 326 a double
square lattice, and Fig. 327 a square framework. In these we see how the choice of lines in a square net fills a whole area with a skeleton, a process which is simply exemplified in


Fig. 328.


Fig. 330.


Fig. 329.


Fig. 331.


Fig. 333.

Fig. 328. Fig. 329 shows a skeleton formed by a square and diagonals, forming interlacing octagons ; Fig. 330, a roof tiling made by the choice of diagonals on a square net ; Fig. 331 a diaper constructed in the same way. Fig. 332 is a triangular
framework formed on a $60^{\circ}$ net ; the upper part is a drop triangular framework, the triangles are only blackened to define the triangles of the design ; the lower part shows a series of interlacing triangles, identified by their blackened centres. Fig. 333 is a hexagonal framework on a $60^{\circ}$ net, with verticals. There are three small hexagons to each triangular form. The network, shown in the lower right-hand corner, being omitted, the lines left form an elaborated skeleton of lines, chosen regularly and uniformly related to one another.


Fig. 334.


Fig. 336.


Fig. 335.


Fig. 337.

Fig. 334 is a framework of interlacing triangles forming a star and outlining a hexagon. Fig. 335 shows a framework in which interlaced stars all fill up the pattern and give a hexagon form ; Fig. 336, a framework of unequal sided octagons, formed on a square net. In Fig. 337 a framework of squares and stars is formed on a square net which is shown in one of them ; the diagonals of the net construct the stars, part only of the diagonals being shown. Figs. 338,339 and 340 show the development of simple line patterns, suitable for band ornament, formed on a network. In Fig. 34I the selection of lines shows
the development of a pattern with the network omitted, and Figs. 342,343 and 344 , the selection of the rectangular framework and added diagonals. Fig. 345 is a plaited band on a $60^{\circ}$ net.


Fig. 338.


Fig. 339.


Fig. 340.


Fig. 34i.


Fig. 342.


Fig. 343 .


Fig. 344 .


Fig. 346.


Fig. 347.

## NNWNNN

Fig. $345^{\circ}$


Fig. 348.

Thus far the skeleton or framework has been composed of lines selected from a network or diagonals of the net, and formed of straight lines only. It remains to add the circle to the materials of construction and to proceed to form frameworks with its aid.

Fig. 346. Semicircles are worked on the square net giving scale work or imbrication, and Fig. 347 shows a scale work on a $30^{\circ}$ net.

Fig. 348 shows a framework on a $45^{\circ}$ net formed of interlacing circles.

Fig. 349 suggests how, on a $60^{\circ}$ network with horizontals, a pattern can be developed from interlacing circles and straight lines outlining a hexagon.



Fig. 350.


Fig. 351.

Fig. 349.


Fig. 352.


Fig. 354 -

Fig. 353.


Fig. 355 .


Fig. 356.

Figs. 350-353 give frameworks of circles on a square net the net of construction being indicated by dotted lines.
Fig. 354 shows the change which takes place in the framework by working with circles on a $45^{\circ}$ net.

Fig. 355 is a skeleton developed from selected semicircles on a square net, and Fig. 356 the same skeleton with additional semicircles which cover the vacant intervals.

Fig. 357 is a complete skeleton of interlacing circles and is a pure circular pattern. This pattern is based on a $60^{\circ}$ net, and it is an example of a simple pattern which a child with a pair of compasses could draw as he fills a paper with circles.


Fig. 357.


Fig. 359.


Fig. 358.


Fig. ${ }^{6} 6$.

Fig. 358 is the result of selecting interlacing circles based on the $30^{\circ}$ net and forms a beautiful geometrical pattern.

Fig. 359 is a scale work on a square net, the radii of the circles being less than the side of the square.

Fig. 360 is an ogee skeleton and Fig. 361 part of a framework, both resulting from the selection of semicircles on a square net.

In Figs. 362 and 363 the selection of quadrants on a square net construct two very different skeletons of interlacing ogee forms.

Fig. 364 is a skeleton composed of circles and semicircles on a square net.

Figs. 365-367 show an effect of combining circles and straight


Fig. 36 x .


Fig. ${ }^{662}$.


Fig. 363.


Fig. ${ }^{664}$.


Fig. ${ }^{665}$.


Fig. 366.


Fig. 367 .
lines on a square net; the net being omitted, a framework or skeleton remains to receive the details of ornament.

Figs. 368-371 show the effect of working on a square net, with circles, semicircles and straight lines.

Fig. 372 is a double scale work formed by concentric semicircles on a square net.


Fig. 368.


Fig. 369.


Fig. 370.


Fig. 372.


Fig. 371.


Fig. 373 .

Fig. 373 is a framework or skeleton border on a square net.

Fig. $374^{\text {is }}$ a framework of quadrants and semicircles with diagonals on a square net.

Fig. 375 is a simple roll border formed by parallel lines and concentric circles on a $45^{\circ}$ net.


Fig. 374.


Fig. 375 .


Fig. 376.


Fig. 377.


Fis. 378.


Fig. 379.


Fig. 380.


Fig. 38 r .

## Units of Pattern.

Having shown how to map out a surface by a network or skeleton, in varied ways, we must now proceed to discuss the units of pattern with which the surface is to be covered. Figs. 376, 377 and 378 are simple circular designs, the construction of which is obvious ; in Fig. 376, the circles are joined by freehand, forming a ball-flower ornament which occurs frequently in 'Decorated' Gothic architecture. Fig. 379 illustrates the variation of pattern caused by emphasising different parts of a design ; the two halves of this unit are really different patterns, though the lines in both are the same. Fig. 380 is a triangular
unit ; for its construction refer to Problem 77, page 49, a triangle


Fig. 382 . drawn to touch the dotted circle at CEG gives the outline. Fig. 38I is an ogee unit constructed on a square and its diagonals. Fig. 382 is a square unit, and Figs. 383,384 , units of intersecting squares. Fig. 385 is a unit constructed


Fig. $3^{8} 3$.


Fig. 38 亿.


Fig. 385.


Fig. 386.


Fig. 387.


Fig. 388.



Fig. 390.


Fig. 39r.

Fig. $3^{8} 9$.
on a 3 by 2 rectangle; Figs. 386, 387, 388 are square units formed by a combination of circles and straight lines. Fig. 389 is a square lozenge, enclosing intersecting circles; Fig. 390 consists of concentric squares and circles, all enclosed in an equilateral octagon. Fig. 391 is a star formed by two interlacing equilateral triangles, with parallel but not equidistant
sides; Fig. 392 is a square star unit ; Fig. 393, a unit of interlacing squares, enclosing a circular design. Fig. 394 is a star hexagon unit. In Fig. 395 geometrical design and freehand are combined on an octagon base.


Fig. 392


Fig. 394.


Fig. 393 .


Fig. 395.

These few examples of geometrical units of pattern must suffice to show the principles of construction. Naturally enough, the scheme of decorative treatment usually leaves the trammels of geometrical design, relying upon the forms of flowers and foliage for the ideas which the repeating patterns carry out. But the object in view has been attained if the student has been led to see, under the intricacies of decoration, the geometrical basis on which it is constructed.

## The Spacing of Walls and other Surfaces.

The wall of a modern dwelling-house is usually divided, for decorative purposes, into the cornice, frieze, field, dado and skirting, as shown in Fig. 396.


Fig. 396.
The various methods for covering such given spaces with ornamentation by means of geometrical patterns are briefly indicated in this section. For the technique of distributing and repeating patterns, suitable for such special spaces, in any decorative scheme, the student should consult any of the various books on Decorative Design.

This chapter aims at teaching him how to construct patterns geometrically：it is quite another and a larger subject which must be separately studied how to make geometrical treatment subservient to the decorator＇s art．

## MTM <br> 

Fig． 397.

## एट्ट

Fig． 400.

Fig． 398.

Fig． 401.

## ？

Fig． 399.


Fig． 402.

## ルけひ』

Fig． 403.

Fig． 405.


Fig． 404.


Fig． 406.


Fig． 407.
（1）．Bands and Borders．
A wall is very often divided into two parts by a horizontal border above the dado；the treatment of such a border is a simple introduction to wall decoration．We begin with a set of bands called＂Greek Frets＂；these are represented in Figs． 397 to 405. They are formed by selecting lines from a square net；an introduction was made to this in Figs．338，339，340，where the lines of construction are seen．In Fig．40I，a raking pattern，the
diagonals of the net are utilized for construction. Figs. 406-408 are intersecting frets, 406 being a Moorish plaited band and 407 an Italian interlacement band; 408 is a chain band, and 409 a


Fig. 408.


Fig. 409.


Fig. 410.


Fig. 4 II.


Fig. 412.


Fig. 413.


Fig. $4 \times 5$.


Fig. 414 .


Fig. 416.


Fig. 417 .


Fig. 418.
straight line band suggesting a series of Maltese crosses formed of onyxes.
With Fig. 4Io a series of circular band ornaments is introduced, leading up to the wave-form introduced in Fig. 414.

The wave and roll form is developed in Figs. 415 to 418 , and its construction carried on progressively to the complicated rolls, Figs. 419, 420. Fig. 42 I shows one of the effects of a


Fig. 419.


Fig. 420.


Fig. 42 I .
double roll combination suggesting to the student what a variety of interesting designs may be evolved from the constructions thus built up. The term " Guilloche" is generally applied to such rolls as are shown in Figs. 419-421.

With Figs. 422-424 is commenced a series of bands having for foundation the combination of straight and curved lines, leading


Fig. 422.


Fig. $4^{23}$.


Fig. 424.


Fig. 425 .


Fig. 426.


Fig. 427.


Fig. 428.


Fig. 429.


Fig. 430.

Fig. 431.



Fig. 434.


Fig. 432.


Fig. 433.
up to Figs. 425,426 , in which the effect of such combinations in decoration is exemplified.

Spiral elements are introduced in Figs. 427 to 430 , and the wave line in Fig. 43I ; and with these materials the beautiful bands given in Figs. 432 to 439 are built up. Figs. 432, 433 and 434 are Greek paintings on terra-cotta. Fig. 435 is a


Fis. 435.


Fig. 437.


Fig. 436.


Fig. $43^{8 .}$


Fig. 439.

French mural painting of the 13 th century ; Fig. 436, a border from a picture by Domenicino (i6th century). Fig. 437 is a Greek terra-cotta, and 438 an Early Gothic French ornament. Fig. 439 is an "æsthetic" design. The two "repeats" in these cases are a sufficient guide to the complete scheme.
(2). Defined Areas-Walls, Cei'ings, Floors, etc.

We come now to deal with the treatment of certain defined areas; hitherto we have only dealt with schemes meant for general areas. Fig. 440 indicates the simplest division of a square so as to give border and corners, while Fig. 44I only


Fig. 440.


Fig. 44 I.


Fig. 442.


Fig. 443.


Fig. 444.


Fig. 445 .


Fig. 446.
affords a plain border, inside which a Maltese cross divides up the space for further ornamentation. Fig. 442 shows a circular centre, and Fig. 443, circular corners. Fig. 444 is a simple indication of the centre and corners due to the inscribed circle, while Fig. 445 shows an elaboration of the centre with wide borders. Fig. 446 is a square design, with a centre which arrests the
eye by being unexpected, as the arms are not radial. Fig. 447 is a square ceiling design ; the top outer circle is completed to show the construction, the true design being given in the lower part. Fig. 448 is a panelling for a ceiling from a tomb in Rome, forming, as in all the cases we have been considering, a skeleton


Fig. 447.


Fig. 448.


Fig. 449.


Fig. 450.


Fig. 45r.


Fig. 452.


Fig. 453.
for decorative treatment. Fig. 449 is a rectangular space which is divided up by selected lines of a square net, with certain diagonals. Fig. 450 is a square lozenge, separating the corners of the rectangle. Fig. 45 I is a cross of St. Andrew, with circular centre, affording triangular panels. Fig. 452 shows a rectangular moulding with subdivision of the space in the
form of a cross. Fig. 453 shows a simple circular treatment of a panel ; Figs. 454, 455, two lunettes with circular treatment. Fig. 456 is a simple lunette and spandrels. Fig. 457 gives a subdivision of a circular space suggesting tracery, and Fig. 458 a trefoil treatment. Fig. 459 gives the complete hexagonal system of circles, inscribed in the given area, from which the artist may select or emphasize symmetric arcs, so as to produce


Fig. 454 .


Fig. 455 .


Fig. 456.


Fig. 457.


Fig. 458.


Fig. 459.


Fig. 460.
an extraordinary variety of design. Fig. 460 is a tracery design, worked from the hexagon of the outer circle. Fig. 461 is the subdivision of an octagonal area, the figure being built on a square net; Fig. 462 on the $45^{\circ}$ net. Fig. 463 is a star figure, which is built up inside an octagon. This is a sample of the many different stars which may be formed by varying the radius of the dotted circle of construction. The student will find a
useful exercise in making several examples, which will illustrate the various designs which result from the change of this circle. For example, if the radius of the dotted circle be made about half of that in Fig. 463, and radii of the larger circle be drawn to the points of the star, we have the familiar appearance of the old mariner's compass as it used to be before the advent of the spider web of the Thomson compass.


Fig. 461.


Fig. 462.


Fig. 463.


Fig. 464.


Fig. 465.


Fig. 466.

Fig. 464 again is a sample of the division of a hexagonal space which is suggestive of many varieties. The elementary feature, the joining of all the points of a hexagon, suggests in appearance the outlines of a transparent icosahedron: a star, inscribed in a circle is formed in the hexagon. The figure itself can be amplified by further outlining the details, the simple plan adopted in the figure is to follow each line with another line parallel to it at a fixed distance throughout. It is evidently possible to produce a large variety of divisions of the hexagon on this model, and the introduction of circular arcs will add still
more. I.
5, 466 the space treated is an equilateral triangle; here a simple plan is to divide it either by a central hexagon, as in Fig. 405, or by a central circle, as in Fig. 466. In each of the examples, the feature of doubled and parallel lines is introduced.
The object which is aimed at in presenting these figures is to suggest modes of setting out defined areas-walls, ceilings, and floors-for decorative treatment. It is this general delineation which is the particular aim of geometrical design. In the completed scheme, no doubt, the framework or scaffolding will be entirely lost, but the aim of the designer is to afford a pleasing arrangement of the space available; and in such a way that the eye should not be arrested by the ornamentation forming a simple network.
It has often been observed that a wall paper for a sick room must not offend in this particular. If a patient's eye is continually challenged by a repeat in network over the surface, or a figure pattern too prominently recurrent over a large area, the mental disturbance is serious. The mind falls to counting the patterns in wearisome persistence, and the more the mind is beyond physical control, the more serious is the effect.

Hence it is desirable that the designer should practise the division of spaces by forms which have a pleasing intricacy of geometrical balance. The methods here indicated are only specimens which should suggest the lines which may be followed, and it is hoped that they will lead the student to exercise his ingenuity in planning more serious designs.

[^1]
## ORNAMENTATION.

## CHAPTER XVII.

## Lettering.

IT is very important for the student to be able to letter his drawings well. The design of Roman lettering is a serious study of which it is possible only to give the merest outline in Figs. 467, 468. The basis of construction is a square of


Fig. 467.


Fig. 468.
rectangular net, with circles to guide in the formation of the serif. The revival of the old style of lettering, with the serif inclined as shown in Fig. 467, letter L, has given a new and interesting impetus to the artistic study of lettering.

These two examples are taken from Albert Durer's Geometrica, in which he gives methods for drawing Roman capitals. Suppose as a groundwork a square. The thick strokes are $\frac{1}{8}$ of the square and the thin strokes are $\frac{1}{16}$ of the square. The serifs are constructed on circles of $\frac{1}{4}$ diameter. Mr. Walter Crane says: "Letters may be sen as the simplest form of definition by means of line. They have been reduced through centuries of use from their pinitive hieroglyphic forms to their present arbitrary and fixed vpes: though even these fixed types are subject to the viriation produced by changes in taste and fancy."

## Shields.

Shields often appear as an element of ornamentation, and must be treated in accordance with the rules of heraldry. It is a very early and general rule that metal must not be placed upon metal, nor colour upon colour; but that they must be placed in contrast. 'Or,' gold, and 'argent,' silver, are the metals used, and 'azure,' blue ; gules,' red ; 'purpure,' 'vert,'. and 'sable' or black are the colours usually employed.

Black may be taken here as indicating metals and white as colours. Furs, ermine and vair, are also used ; but of these ornamentation takes no account. Figs. 469 to 472 show the principal divisions of shields. Fig. 469, checquy, a shield divided


Fig. 469.


Fig. 470.


Fig. 47 r.


Fig. 472.
in chequers or small squares like a chessboard ; the number varies. Fig. 470, quarterly, the field being divided into four quarters. Fig. 47 I , the Pale or a vertical strip set upright in the middle of the shield and one-third of its breadth. Fig. 472 represents a band division. The upper part is the chief, occupying one-third of the height, the fess is the bar, horizontally placed in the middle of the field.

Among the other divisions of the field must be reckoned the chevron, a $\Lambda$-shaped strip and the cross, usually a Greek cross of equal arms. When plain, this cross is in breadth one-third of the shield; but its varieties are manifold.

## Diaper, Chequer, Spot, Powder.

ct diaper is a repeated pattern covering a given surface intervals; for example, Fig. 473, which gives the appear-


Fig. 473.


Fig. 474.
encaustic tiling with no variation of pattern ; Fig. 474 $i$ i. an c imple of a chequer, which consists of a repeat alternately with $v$ :ant spaces.

Fig. 475 is a spot pattern ; it must be observed that the spot pattern has large regular vacant intervals, the chequer, a vacant space equal to that of the repeat.


Fig. 475.

Fig. 477.

Fig. 479.



Fig. 476.


Fig. 48 .

Powder again differs from spot in point of scale, tlre unit of powdering should be small and simple. .Powdering consists of small and insignificant units of repetition and may be combined with spot. For example, Fig. 476 is a combination of spot and powdering.

Fig. 477 is a stripe and band pattern. Units of pattern when arranged in narrow lines are called 'stripe,' and when wider, 'band.' For instance, Fig. 478 is the elementary stripe and band, the simplest form of this decoration. Fig. 479 is a
chequered band, with stripes arranged for diagonal decoration ; Fig. 480 is a panelling, derived from bands, and Fig. 481 a piece of parquet flooring. Fig. 482 is an application of diaper or chequer in more elaborate form. It represents an inlaid work of independent, interlacing squares. In Fig. 483, crosses are


Fig. $4^{81}$.


Fig. $4^{83}$.

Fig. 482.



Fig. $4^{84}$.
arranged as a diaper for mosaic decoration. Fig. 484 is a chequer formed by a combination of Greek fret and square foliage ; Fig. 485 is a more elaborated diaper or spot pattern on a circular basis and constructed on a $45^{\circ}$ net, which forms part of the design. Fig. 486 is a diaper pattern formed in marble mosaic, from San Vitale, Ravenna. Fig. 487, a ceiling panelling


Fig. 485.


Fig. $4^{86}$.


Fig. 487.
conveying the impression of spot and powdering. It is taken from a mediæval enamel in Cologne. Fig. 488 is a mosaic


Fig. 488.
flooring of simple construction and a charming intricacy. Fig. 489 is one panel of ceiling decoration, in which only one


Fig. 489.
repeat is shown. Fig. 490, a scale-work diaper, a simple construction if based on a $45^{\circ}$ net. Fig. 491, a very pleasing combination of square and circular treatment suitable for a mosaic flooring. Fig. 492 is a reproduction of a Byzantine bas-relief from the Cathedral of San Marco, Venice. It is


Fig. 490.


Fig. 491.


Fig. 492.
a beautiful example of varied interlacing. The constructions are extremely simple, and the effect is due to the unexpected,


Fig. 493.
which piques the interest. Fig. 493 is an Egyptian ceiling decoration ; in reality it is a simple diaper and spot with spiral


Fig. 494.
construction. Fig. 494 is the corner of a rectangular mosaic. Fig. 495 is an interlacement band ornamenting a northern MS. of the eighth or ninth century ; Fig. 496, a geometrical band decoration in coloured marble from the wall of Mackworth Church, Derbyshire. Fig. 497, a band of Moorish mosaic from

Granada, which, it may be observed, is interchangeable, the black and white spaces being exactly equivalent.


Fig. 495.
Decoration based on geometrical construction is of infinite variety, and a thorough familiarity with geometrical relations will regulate genius and inspire the designer with a just view of


Fig. 496.


Fig. 497.
those forms which will be pleasing and restful to the eye, as well as satisfying to the natural demand for relief.

## ARCHITECTURE.

## CHAPTER XVIII.

## Arch Forms and Tracery.

The semicircle was the first arch-form to appear in building. In the earliest times (Nippur, B.C. 4000), such arches were employed below the level of the ground. The tendency of the round arch to sink when bearing any weight led these early builders only to use it with that strong lateral support. The Romans employed it largely ; as their bridges and aqueducts standing to this day exemplify. The Pont du Gard, near Nîmes (B.c. 19), is about 160 ft . high and 880 ft . long ; built of


Fig. 498.


Segmental
Fig. 499.


Eilijstic
Fig. 500.
large stones without cement, it affords a striking example of the stability of the semicircle.

The round arch may be semicircular, as Fig. 498 ; segmental, as Fig. 499 ; "elliptıc" or three-centred, as Fig. 500. It may be "stilted," as Fig. 50I, the semicircle being continued in straight
lines ; or a "horse-shoe," as Fig. 502, the circle itself being continued. This last is a Moorish feature, the semicircular a Roman, and the stilted arch a Byzantine feature.

The Basilica, or Hall, originally a place of business, became in the hands of the Roman builders a structure of heavy round


Fig. 501.


Fig. 502.
arches and circular windows. This form was adopted for the early Christian churches ; we have such a building at Brixworth, Northants.

The Norman builders (io66 to i190) followed up the semicircular brick arches, building them in stone ; the heavy round pillars with cushioned capitals, and the heavy arches with geometrical ornamentation being the sucressors of the Roman work.

The origin of the pointed arch has been much discussed ; an example is found in Cairo, a horse-shoe arch of the ninth century; in fact, at that time, this form was regarded by the Moslems as their special emblem. It is met with in the Crusaders' churches throughout the twelfth century. If in England it has been developed from Norman arcading, as is often supposed, there is an example of the process on the towers of Southwell Minster. There is a semicircular arcading ; then on another face, semicircular arcades intersect; the lancets thus formed become windows in the next, and, finally, the arcading disappears and lancet-headed windows are seen alone.

The Transitional Period, heavy Norman work with pointed arches (about II40 to 1200), is well exemplified in the arcading of St. John's Church, Chester ; best, however, in the Choir of Canterbury Cathedral.

The Early English, or Lancet Style (about I 190 to I 300), was thus developed at the end of the twelfth century from the circular or Romanesque ; the lancet windows (Fig. 503) and the clustered pillars (Fig. 543) giving elegance and lightness. Characteristic ornament was introduced, and a style grew up which is essentially English.

At this time also Tracery begins to appear, apparently developed as follows. Lancet windows are placed together ; in Salisbury Cathedral, our great instance of a complete Early English building, are seen combinations of two to even seven lancet windows together. Again, three windows are often placed


Ianca
Fig. 503.


Fig. 504.
together, the middle window being higher than the other two. The three would be treated as one window, one arch moulding including them all. Again, two lancet windows are placed close together with a trefoil or quatrefoil (Fig. 512) above them, and are treated as one window ; in Westminster Abbey, a small triangle appears besides the small quatrefoil. This "wall tracery" led to "plate tracery," where the wall is thinned to a


Trop
Fig. 505.


Segmental poinded.
Fig. 506.
single piece of stone ; and plate tracery led to "bar tracery," where the wall between windows becomes a bar, and the quatrefoil becomes a geometrical design in the arch.

This leads to the Geometrical Period (about 1260 to 1320), and, at this point, the study of tracery begins. The window is divided, according to its size, by a number of vertical bars or "mullions" ; the arch is equilateral (Fig. 504) and filled with circles, trefoils and curved triangles in strictly geometrical design. The "element" of the design (Fig. 509) is the skeleton showing the centres of the circles of the tracery. The element of some windows is nothing but a $45^{\circ}$ network, with quatrefoils as a unit of pattern. The element (Fig. 509) will form an interesting exercise for the student to complete, and the ruined window at the end of the book (p. 262) will be found a more difficult problem of the same sort. In the Geometrical Period, circular foils (Fig. 512) alone are used ; in the last quarter of the thirteenth century, pointed foils (Fig. 513) are introduced.

The Decorated Period (about 1300 to 1375). The simplicity of style, so far described, led, in the hands of ingenious designers, to flowing curves, such as Figs. 5II, 515, 518 suggest. This flowing style is a purely English development ; it may be called Flowing Decorated (about 1315 to 1360). The "ogee" arch (Fig. 507) and the pointed foiled arch are introduced; also


Fig. 507.


Four centred
Fig. 508.
bands of wavy foliage of a natural design. But this leads us beyond the scope of Geometrical Design.

It seems, however, that the excess of ornament led to a return of simpler forms in the last quarter of the fourteenth century, and introduced the Perpendicular Style (about 1400 to 1545). This deserves our attention because of its geometrical character.

Straight lines are the feature of this style in place of the flowing lines of the later Decorative Period, circular lines and circular cusping. Horizontal transoms are introduced into the large windows at this period, and become a decorative feature. The


Fig. 509.


Fig. ${ }^{510}$.


Fig. 51 It .
enclosing square which is used for construction in the element 509 becomes the outline of door and window. The four-centred arch is due to this period (Fig. 508).

This, again, is a style which is peculiarly English, and is more appreciated nowadays. Perhaps it is too mechanical, and


Fig. ${ }^{512}$.


Fig. ${ }^{113}$.
there may be too much repetition of ornament; but to this period belong, the great East Anglian churches, and such masterpieces as the Tower, Choir, and Lady Chapel of Gloucester

Cathedral. It forms a grand close to the development of our English Church Architecture.


Fig. ${ }^{514}$.


Fig. 517.


Fig. 515 .


Fig. 516.


Fig. 518.

## Greek and Roman Mouldings.

Mouldings have been called the alphabet of architecture ; they are the elements which determine and give expression to the parts of a building.
The simple forms of moulding are :
The Roman ovolo, a quadrant (Fig. 519).


Fig. 519.


Fig. 520.


Fig. 521.

The cavetto, or holiow (Fig. 520), which is the reverse of the ovolo.

The torus, or half-round (Fig. 52I).

Other mouldings and cornices or other designs can be made by arranging these with flat spaces, above, below, or between them.

A fillet (Fig. 522) is a small flat face, and the torus when small is called a bead or astragal (Fig. 523), which may be


Fig. 522.


Fig. 523.


Fig. 524.
incised, so as not to project from the flat surface. Severai parallel beads together are called reeding.

The cymia recta (Fig. 524) is formed by combining ovolo and cavetto, the hollow being uppermost, and is suitable for a cornice.

The cyma reversa, or ogee (Fig. 525), a similar combination, with the hollow at the bottom, is suitable for a base moulding.

The scotia is formed by two quadrants, as shown in Fig. 526.


The Greek mouldings correspond with these, but their section is not circular like the Roman. They, are for the most part constructed with conic sections, viz. ellipses, or parabolas. But in all probability they were drawn in by hand.

Some construction lines for these mouldings are shown in the figures, 527, ovolo; 528, cyma recta; 529, cyma reversa or ogee ; 530, scotia.

The Bird's Beak or Hawk's Beak moulding (Fig. 531) is common in Greek Doric architecture. It is a cyma recta surmounted by a heavy ovolo which casts a bold clear shadow


Fig. 529.


Fig. 530.


Fig. 531.
over the cyma. It is particularly interesting, because it disappears from architecture entirely after the best period of Athenian art.

Greek architecture is distinguished by the grace and beauty of its mouldings; it has been remarked that their sections are mostly elliptic. They are, however, not regular curves; they inust be drawn, rules cannot be given for describing them. Symmetry, proportion, and refinement are the characteristics of Greek ornament.

The mouldings of Roman origin are in general form the same as the Grecian, but their contour is bolder and section circular.

The ornamentation of Roman mouldings was no doubt borrowed from Greece, but it is less restrained. Roman architecture is overdone with ornamentation; foliage and various subjects in relief covering every moulding and surface.


Fig. 532.


Fig. 533.

One or two characteristic specimens of ornamentation may be given. Fig. 532 is the egg and tongue or arrow on the Greek echinus or ovolo; and Fig. 533 the egg and dart on a

Roman ovolo. A bead or astragal may be divided up, as Fig. 534. Figs. 535 and 536 are typical ornamentations of torus moulding.


Fig. 534.


Fig. 535.


Fig. 536.

Fig. 537 is a Greek leaf ornament for cyma reversa from the Erechtheum at Athens, and Fig. 538 a Roman leaf ornament for the same type of moulding.


Fig. 537.


Fig. 538.

## Gothic Mouldings.

Mouldings were developed contemporaneously with the other features of Gothic architecture. In the Norman period, as might be expected, these are square and circular in section.

The stock Norman moulding (Fig. 539) consists of a broad hollow surmounted by a broad fillet, from which it is cut off by a small sunk channel ; in fact, the hollow is set off by 'quirks' or returns.


A plain round projection is frequently found with a narrow fillet above it, the quirks being chamfered (Fig. 540). This moulding is called a bowtel.

There are very deep mouldings over the round Norman doorways, consisting simply of squares and circles, at Iffley, Oxford, for example (Fig. 541).


Fig. 54 I.


Fig. 542.

The Early English architects developed this style of moulding, retaining circular forms and almost entirely eschewing ogee or reversed curves. A single specimen only need be given, part of a doorway, at Woodford, Northants (Fig. 542).

It may be noticed here that fillets are freely run down the face of circular mouldings, that each circle is defined, and that there is no returned curve. These may be taken as the simple characteristics of Gothic mouldings.

## Gothic Piers.

One architectural feature which should be mentioned is the


Fig. 543 . development of the simple circular and square pier of the Norman style into the elaborate piers of the Perpendicular period.

The massive Norman pier in the hands of the early English builders was lessened in size and had shafts set round it, as Fig. 543, a specimen from Salisbury Cathedral or the north transept of Westminster Abbey. The shafts increased in number and were incorporated in the body of the pier, still preserving a circular contour.

The contour became of a lozenge plan in the style called Decorated, and the number of pillars is much increased. The shafts were arranged diamond wise, so many as would stand close together, with only a fillet or small hollow between them.


Fig. 544 .


Fig. 545 .

Fig. 544 shows a Decorated pier at Dorchester, Oxfordshire, and Fig. 545 a Perpendicular pier from Rushden, Northants.
(For Exercises see p. 235.)

## MISCELLANEOUS EXERCISES

## FROM EXAMINATION PAPERS OF THE BOARD OF EDUCATION

## ORTHOGRAPHIC PROJECTION (Сн. XIV.).

1. A plan and elevation are given of a buttress projecting from a wall (Q. I). Draw a fresh elevation on a vertical plane which makes an angle of $45^{\circ}$ with the plane of the wall.

Q..

Q. 2.

Q. 3.
2. The end elevation is given ( Q .2 ) of a small cofter or caddy, the length of which is to be $3^{\prime \prime}$. The lid has four sloping faces, which all make the same angle $\left(30^{\circ}\right)$ with the horizontal. Draw the plan of the lid.
3. The plan is given (Q. 3) of an octagonal tray or dish, the height of which is $\frac{1}{2^{\prime \prime}}$. Make an elevation on the given $x y$. Only the visible lines need be shown in elevation, and the thickness of the material is to be neglected.
4. The diagram (Q. 4) represents a doorway in a wall, the door being shown opened at an angle of $45^{\circ}$ with the surface of the wall. Draw an elevation of the door when closed, i.e. showing the true form of the panels. Only the door need be drawn, not the surrounding mouldings. Your construction must be shown.

5. The plan is given (Q. 5) of a square slab, $a b$ being one edge of a square base. Draw an elevation on a vertical plane at right angles to $a b$. Show in plan the section made by a horizontal plane containing $a b$.

Q. 6.

6. The plan and end elevation are given (Q. 6) of a simple hut. Draw an elevation on a vertical plane which makes an angle of $30^{\circ}$ with the long walls of the hut. (N.B.-Only the visible lines need be shown.)
7. The diagram (Q. 7) shows an elevation and section of an opening in a wall. Make a second elevation when the face of the wall makes an angle of $45^{\circ}$ with the vertical plane of projection.
8. The diagram ( Q .8 ) shows an elevation of a square slab, $A B$ being one side of a square face, $3^{\prime \prime}$ long. Draw the plan of the slab, and show a cylindrical hole of $2^{\prime \prime}$ diam. pierced through its centre.

9. The plan is given (Q. 9) of a square prism, of which $A B$ represents a square face, $3^{\prime \prime}$ each side. Determine the elevation of the prism on the given $x y$, and add the elevation of a circular hole of $2^{\prime \prime}$ diameter piercing the centre of the prism.
10. The diagram ( Q . 10) shows a perspective sketch of part of a buttress. Make an approsimate sketch plan and a side elevation.

## SECTIONS OF SOLIDS (Сн. XV.).

1. Make an approximate sketch plan, and also a sectional elevation, of the mortar of which a perspective sketch is given (Q. I), assuming

Q. 1.

Q. 3.
that the inside form is a hemisphere. Show clearly any construction you would suggest. The size of your drawing should be about 3 times that of the diagram.
2. A right cylinder of $2^{\prime \prime}$ diameter is cut by a plane, making an angle of $30^{\circ}$ with the axis of the cylinder. Show the true form of the section.
3. The plan is given (Q. 3) of a right cone (diameter of base, $2 \frac{1_{2}^{\prime \prime}}{2}$ ), of which $V$ is the vertex. Make an elevation on the given $x y$, and show the section by a vertical plane parallel to the base and $\mathrm{I}^{\prime \prime}$ from it.

## DESIGN.

## CONSTRUCTION LINES AND UNITS, pp. 184-195.

1. Sketch four illustrations of ornament formed by circles, and explain the object they fulfil in certain cases.
2. Draw clearly, with instruments or freehand, the geometrical basis on which the given "diaper" pattern (Q. 2) is constructed. Plan the scale of your diagram to show two "repeats" of the pattern in a width of $3 \frac{1}{2}^{\prime \prime}$. Show 5 or 6 repeats in all. (Only sufficient of the ornamental detail need be sketched to indicate its position.)

Q. 2.

Q. 3
3. Sketch, with instruments or freehand, one unit of the given diaper (Q. 3), showing clearly your method of setting out its details. Make your drawing about three times as large as the diagram. [The height and width of the figure are equal.]
4. Draw, freehand or with instruments, a system of construction lines on which the given system of quatrcfoils ( $Q$. 4) can be built up. Show how you would determine the centres of the arcs, and the points of contact. Three or four repeats of the unit should be indicated about twice the size of the diagram.
5. Show clearly any geometrical construction you would think useful in setting out the given "Tudor rose" (Q. 5) about four times the dimensions of the diagram. You need only sketch so much of the flower as is needed to illustrate your construction.

Q. 4.

Q. 5 .
6. Draw, with instruments or freehand, the system of construction lines on which you would build up the given repeating pattern (Q. 6). Show also one complete unit of the repeat.

7. Show clearly how you would set about drawing the given pattern (Q. 7). Mark what you consider the unit, and indicate any constructions you think needful. Only two complete adjacent units need be shown, about four times the scale of the diagram.
8. Any triangle can be repeated so as to cover a space without leaving interspaces. Show how this can be done, and sketch four other shapes, rectilinear or curved, which will repeat in a similar way.
9. A floor has to be covered with tiles which are squares and regular octagons in shape. Sketch the pattern so formed, showing clearly how you would set it out, and marking the unit of repeat.

## SPACING OF SURFACES, pp. 196-201.

1. Draw a Greek fret without keys, but with a border at top and bottom ; the fret, borders and spaces, to be $\frac{18^{\prime \prime}}{}$ wide. Also another fret with a tee and a border top and bottom ; borders, spaces and fret each $\frac{1^{\prime \prime}}{8}$ wide. Explain the principles of the ornamentation and the surfaces to which they can be applied.
2. Draw three sorts of frets, and another fret on the slant, and an instance of frets alternating with ornamented panels, each $\mathrm{I}^{\prime \prime}$ high. Explain the principles and the surfaces to which they are properly applied.
3. Draw a band ornamented with square panels filled in with some usual Greek ornament, divided from one another by Greek keyed frets, meeting at centre line of panel, with border top and bottom: height exclusive of border $I^{\prime \prime}$, length $3 \frac{\frac{5}{2}^{\prime \prime}}{8}$, to contain three panels. Explain the principles of the ornamentation.

4. Make a drawing of one unit of the given border (Q. 4), increasing the length of the "unit" to $2 \frac{1}{4}$ ", and the other dimensions in proportion. Indicate the method by which the figure should be constructed.
5. Make an enlarged copy of one unit of repeat of the given border (Q. 5), the height of your drawing being increased to $\mathrm{I}_{\frac{1}{2}}$ and the length in proportion. Show a construction for obtaining the divisions of the circle.
6. The diagram (Q. 6) shows half a circular plate. Indicate a method by which the leading wave line of the ornament could be made up of arcs of circles of equal radius.

Your drawing should be three times the diagram.
7. Show the construction you would employ in setting out the given border (Q. 7).

Not more than three repeats should be drawn.

Q. 7.

## DEFINED AREAS, pp. 202-206.

1. Show the construction lines upon which the ornament in the square panel (Q. I) has been designed.
2. The figure ( $Q .2$ ) shows one quarter of the decoration of a circular plaque. Complete the circle and set out the panels. The position only of the freehand ornament need be indicated.

3. Show how you would proceed to modify the given figure (Q. 3) so as to make the central panel a regular octagon, the width of the four side panels remaining unchanged.
4. What is the geometrical basis of the given design (Q. 4) for chipcarving? (N.B.-Show only the main lines. Do not attempt to copy the figure completery.)
5. Draw a coffered ceiling, $6^{\prime \prime}$ square, with beams round the outside, with a circular panel in the middle, and four angle or spandrel panels, the spacing to be in harmonic proportion. Ornament the coffers and the soffits of the beams if you can, and explain the principles of the ornamentation.

## ORNAMENTATION, pp. 207-215.

1. Draw, with instruments or freehand, two different arrangements by which the two given ornaments ( Q , 1) may be used, alternating with one another, so as to form a "diaper" pattern.
(The ornaments may be roughly sketched simply to indicate their position.)
2. Show how, by repeating and reversing the given lines (Q. 2), an " all over" pattern may be obtained. Indicate the lines of construction.

Q. т.

Q. 2.

Q. 3 .

Q. 5.
3. Sketch, with instruments or freehand, an "all over" diaper pattern formed by repeating the given unit (Q. 3). Show nine repeats of the unit, with the leading lines of the construction. Make each unit about three times the size of that in the diagram. (The height and width of the figure are equal.)
4. The outline of the diaper pattern formed by placing repeats of the given figure (Q. 3) in contact with one another is made up of semicircles. Draw at least four repeats of the outline so as to show clearly where centres and points of contact of the semicircles occur. Make each unit about five times the size of that in the diagram.
5. The diagram (Q. 5) represents a stencilled ornament which it is desired to repeat so as to form a "diaper" pattern. Sketch two ways in which this can be done, the repeats of the unit being placed adjacent to one another. Show four repeats in each case about twice the size of the diagram.
6. Indicate clearly a geometrical basis for the given repeating pattern (Q. 6), and show what you consider to be the unit. (The freehand ornament should only be shown once.)
7. It is desired to restore the complete circular ornament of which a fragment is shown (Q. 7). How would you do this?

The restorations of the dark portions should be disregarded altogether in your drawing.

Q. 6.

Q. 7.

Q. 8.
8. Indicate any geometrical construction you think desirable in setting out the given pattern (Q. 8).

Show clearly what you consider the unit of the pattern. (N.B.-Do not try to copy all the details; only show enough to make your meaning clear.)
9. Draw, with instruments, specimens of scale work (imbrication), showing two scales and a half to each in length, each scale to be $\frac{3_{8}^{\prime \prime}}{8}$ wide. Give specimens of scales formed of half circles, oblongs with rounded ends, outlines of leaves, leaves with an outer margin and ribs, and one whose leaves are double ogees with the point of the leaf turned up, and with ribs, and one of trefoils with margin and filled with ornament. State to what surfaces they can be properly applied.

ARCHITECTURE, pp. 216-225.

1. Draw the given diagram of window tracery (Q. I), using the figured dimensions. The arch is "equilateral."

Q. I.

Q. 2.

Q. 3.
2. Draw the given outline of window tracery (Q. 2), using the figured dimensions. The arch is "equilateral" and all the arcs are of equal radius.
3. Draw the "cyma recta" moulding shown (Q. 3), adhering to the given dimensions. The curve is composed of two quarter-circles of equal radii, tangential to one another and to the lines $A B$ and $C D$ respectively.
4. Draw the "scotia" moulding shown (Q. 4). The curve is made up of two quarter-circles of $\mathbf{r}^{\prime \prime}$ and $\frac{1}{2}$ " radius respectively.

Q. 4.

Q. 5 .

Q. 6.
5. Draw the "rosette" shown (Q. 5), according to the figured dimensions.
6. Draw the "ogee" arch shown (Q. 6) to a scale of $2^{\prime}$ to $\mathrm{I}^{\prime \prime}$. The arcs are all of $2^{\prime}$ radius. The methods of finding the centres and points of contact must be clearly shown.
7. Draw the moulding shown ( Q .7 ), adhering strictly to the figured dimensions. The arc of $\frac{1^{\prime \prime}}{}$ radius is a quadrant.
8. Draw the "cyma recta" moulding shown in the diagram (Q. 8), using the figured dimensions. The curve is composed of two equal tangential arcs each of $\frac{3 \prime \prime}{4}$ radius.
9. Copy the cornice given (Q. 9), increasing the total height to $2 \frac{3}{4}$ ", and the other measurements in proportion. You may draw the "cyma" moulding by any geometrical construction that seems to you suitable.

Q. 7.

Q. 1 .

Q. 8.

Q. II.

Q. 9.

Q. 12.
10. How would you draw the leading lines of the window tracery given (Q. Io)? (N.B.-Do not try to copy the whole figure, but only clearly indicate your method.)
11. Show how to set out the given figure (Q. if).
12. Make an approximate sketch plan or elevation (but not both) of the given column base (Q. 12).

# EXAMINATION PAPERS IN GEOMETRICAL DRAWING. 

BOARD OF EDUCATION.

## GENERAL INSTRUCTIONS TO CANDIDATES.

You may not attempt more than five questions, of which three only may be chosen from Section A, and two only from Section B. But no award will be made to a Candidate unless he qualifies in both sections.

All your drawings must be made on the single sheet of drawing paper supplied, for no second sheet will be allowed. You may use both sides of the paper.

None of the drawings need be inked in.
Put the number of the question close to your workings of problems, in large distinct figures.

The value attached to each question is shown in brackets after the question.

A single accent ( ${ }^{\prime}$ ) signifies feet ; a double accent (") inches.
Questions marked (*) have accompanying diagrams.
Your name may be written only upon the numbered slip attached to your drawing paper.

## I.

## SECTION A.

In this section you may attempt three questions only, SHOWING YOUR KNOWLEDGE by THE USE OF instruments.

The constructions must therefore be strictly geometrical, and not the result of calculation or trial.

All lines used in the constructions must be clearly shown.
Set squares may be used wherever convenient. Lines may be bisected by trial.

1. Draw a diagonal scale $\frac{1}{11}$ of full size, by which feet, inches, and eighths of an inch may be measured up to $5^{\prime}$.

By means of this scale construct a triangle having its altitude $2^{\prime} 2 \frac{1}{4}^{\prime \prime}$, one side $2^{\prime} 5 \frac{7^{\prime \prime}}{3}$, and base $2^{\prime} 0_{\frac{5}{8}}^{\frac{5}{8}}$. Write down the length of the third side.
*2. Draw the given figure by inscribing seven equal squares within a circle of $1 \frac{7}{8}^{\prime \prime}$ radius.
(20)

*3. Copy the given guilloche ornament, making the radii of the circles $\frac{1^{\prime \prime}}{}{ }^{\frac{1}{2}} \frac{1}{2}^{\prime \prime}, \frac{3^{\prime \prime}}{4}$, and $\mathrm{I}^{\prime \prime}$ respectively.
4. Describe two circles of $\frac{5_{8}^{\prime \prime}}{8}$ radius touching each other, and one of $I^{\prime \prime}$ radius touching the first two. Then describe a fourth circle touching all these three. All constructions must be clearly shown, and all contacts indicated.
(20)
5. A quadrilateral $A B C D$ is to be described about a circle of $\frac{7 \prime}{8}$ radius. $A B=2 \frac{5^{\prime \prime}}{8}, A D=2 \frac{1^{\prime \prime}}{2}$, and the angle $B A D$ is $42^{\circ}$. Draw the figure, show all points of contact, and write down in degrees the angle $A D C$. Then draw a similar quadrilateral having the radius of its inscribed circle $\mathbf{I}_{\frac{3}{5}}$.
*6. An elevation is given of part of an octagonal pillar with square base. Draw its plan, and an elevation on a vertical plane which makes $30^{\circ}$ with one of the vertical faces of the base.
(20)
*7. The diagram shows the elevation of a solid composed of a cylinder capped by a portion of a sphere. Draw the true form of the section made by the plane indicated by the dotted line.
(20)

## SECTION B.

In this Section you may attempt two questions only.
All freehand work employed in this section must be neat and careful, and its intention must be made quite clear.

All constructions must be clearly shown.
*8. Redraw the given pattern, with instruments, altering the proportions so as to make the octagons regular and of $\frac{3^{\prime \prime}}{4}$ side.

Q. 8.
*9. Show the geometrical framework on which the given pattern is based. Show clearly what you consider the unit of repeat, and draw four repeats, sketching only enough of the pattern to make your intention plain.
*10. Draw the geometrical constructions you think necessary in setting out the plate, making your drawing double the dimensions of the print. It will suffice if one quarter of the design is clearly shown.
*11. Draw, using instruments, one of the cusped arch-forms, preserving, as accurately as you can, the proportions of arch and cusping. Make your drawing twice the dimensions of the print.
(20)
*12. Make an approximate sketch plan and front and side elevations of the flower-holder, using instruments where you think advisable. Arrange your drawings so that one is projected from another.

Q. 9 .

Q. 1 .

Q. 12.

## Il.

## SECTION A.

1. A drawing made to a scale of $\frac{1}{8}$ of full size has to be re-drawn so that the dimensions shall be enlarged by one-fifth. Make a scale for the new drawing, showing feet (up to $3^{\prime}$ ), inches, and (diagonally) eighths of an inch.

Figure the scale properly, and show by two small marks on it how you would take off a distance of $\mathrm{I}^{\prime} 48^{\frac{5^{\prime \prime}}{}}$.

*2. Copy the given figure, making the diameter of the outer circle $3 \cdot 8^{\prime \prime}$. Show how to determine all the points of contact between circles and straight lines.

Q. 6.

Q. 7.
3. Two points, $A$ and $B$, are $\mathrm{I}^{\prime \prime}$ apart. Find a third point, $C, \mathrm{I} \cdot 6^{\prime \prime}$ from $A$ and $2^{\prime \prime}$ from $B$. With centre $A$ and radius $A B$ describe a circle. Describe a second circle touching the first at $B$, and passing through $C$. Describe a third circle of $\frac{1_{2}^{\prime \prime}}{}$ radius touching the first two (but not at $B$ ).
4. The foci of an ellipse are $3^{\prime \prime}$ apart. A point $D$, on the curve, is $\mathbf{I}^{\prime \prime}$ from one focus and $3^{\prime \prime}$ from the other. Draw the ellipse and a normal to it at $D$.
(16)
*5. The point of intersection of the two given lines being inaccessible, draw through the point $P(\mathrm{I})$ a line which would pass through the point
of intersection of the two given lines, and (2) a line making equal angles with the two given lines.
*6. The diagram shows the plan of a right square pyramid. Draw an elevation.
*7. The elevation is given of an "elbow" formed by two pieces of cylindrical piping. Draw the plan, and also the true form of the intersection of the pipes. The thickness of the material may be neglected.
(22)

## SECTION B.

*8. Show how you would set out a geometrical framework for the given pattern, so as to exhibit a number of repeats. Only enough of the

Q. 8.
ornament should be sketched to show quite plainly what you consider the unit of repeat.
9. Regular pentagons will not by themselves cover a surface. Draw any form which in combination with regular pentagons would serve this purpose. Make a diagram of the pattern formed, and show clearly what you would use in practice as the unit of repeat.

Q. $\frac{1}{}$
*10. Show what geometrical aids you would employ in drawing the square panel given. Make your drawing about the size of the figure.
*11. What geometrical means would you use in setting out the fan shown? Make your drawing about the size of the figure.
(16)
*12. Draw a plan of the given table, showing clearly any constructions you would use.

Q. ir.

(2). 12 .

## III.

SECTION A.

1. Make a plain scale, to show feet and inches up to 5 feet, on which a distance of $3^{\prime} 6^{\prime \prime}$ is represented by $4 \frac{1}{4}^{\prime \prime}$. Finish and figure the scale neatly and carefully.

Draw to this scale an oblong $3^{\prime} 3^{\prime \prime} \times 2^{\prime} 5^{\prime \prime}$, and in the centre of it place a second oblong of the same shape but having its longer sides $2^{\prime} 9^{\prime \prime}$. Measure the breadth of this smaller oblong to the nearest halfinch.

*2. Complete half the given figure, which is made up of regular pentagons. Make the radius of your enclosing circle $2^{\prime \prime}$. Show clearly any construction you employ.
*3. Copy the given figure, making the radius of the outer circle $1 \frac{3_{4}^{\prime \prime}}{4}$, and that of the inner one in proportion.
*4. Draw the given figure. The curves are to be composed of arcs of circles of $0.5^{\prime \prime}$ and $\mathrm{I} \cdot 5^{\prime \prime}$ radii.
5. Two straight lines, $A B$ and $C D$, are $3^{\prime \prime}$ and $3 \cdot 5^{\prime \prime}$ long respectively. $A$ is $\mathrm{I} \cdot 5^{\prime \prime}$ from $C$, while $B$ is $\mathrm{I}^{\prime \prime}$ from $D$ and $3 \cdot 5^{\prime \prime}$ from $C$. Describe two circles each touching both $A B$ and $C D$, one passing through $A$, the other through $B$.
*6. The outline of a "scotia" moulding is shown. If two lengths of this moulding are " mitred" together at right-angles, show the true form of the cut surface of the mitre.
*7. A lamp-shade, in the form of a truncated regular hexagonal pyramid, is made of six pieces of card, of the exact shape and twice the size shown. Draw its plan and elevation in any position.

## SECTION B.

*8. Show, using instruments, how you would set out the geometrical framework of the given openwork panel. None of the "cusping" need be drawn.

Q. 8.
*9. Draw neatly with instruments the framework or "net" of the given pattern. Show clearly what you intend to be the unit of repeat, and finish not less than four of these so as to show repeats both in width and height.
*10. Set out, as nearly as you can, the construction needed for the geometrical part of the ornament round the semicircular door-head of which about half is shown. Only one unit of each ornament need be completed. Make your drawing twice as large as the diagram.

Q. 9 .


Q $\frac{1}{}$.
*11. Redraw the given figure, altering the proportions of the parts so that the sides of the four corner panels shall be exactly half those of the centre panel. Make the side of the outside square $3^{\prime \prime}$, and the margins throughout $0.2^{\prime \prime}$ wide.

Q. II.
*12. Draw an approximate sketch-plan, with front and side elevations, of the given steps. Arrange your drawings to show how one is projected from the other.


## IV.

## SECTION A.

1. Six feet are represented on a drawing by one inch. Make a scale for the drawing by which single feet can be measured up to $40^{\prime}$, and show inches diagonally.

Figure the scale properly, and show by two small marks on it how you would take off a distance of $20^{\prime} 8^{\prime \prime}$.

Q. 2.

Q. 3.
*2. Make a copy of the diagram, using arcs of $I \frac{1^{\prime \prime}}{}$ and $\frac{5^{\prime \prime}}{8^{\prime}}$ radii only. Show clearly how all points of contact are obtained.
*3. Copy the diagram, making the radius of the outer circle $1{ }^{\frac{3}{4}}{ }^{\prime \prime}$.
*4. The diagram shows a symmetrical figure composed of straight lines and five semicircles of equal radii. Draw a similar figure having a total height of $3 \frac{1}{1 "}^{\prime \prime}$.

Q. 4.

Q. 6.
5. Construct a regular nonagon of $1 \frac{3 \prime \prime}{4 \prime}$ side. Describe a circle touching all the sides of the nonagon. Within the circle inscribe a regular nonagon having its sides parallel to those of the first one.

If you employ a protractor for measuring an angle, this must be clearly shown and the number of degrees stated.
*6. The diagram shows the elevation of a short prism, or slab, the bases of which are equilateral triangles. Draw the plan, and write down the angle which the bases make with the vertical plane of projection.
7. A right cone, diameter of base $3 \frac{1}{2}^{\prime \prime}$, height $2 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$, has the plane of its base inclined at $45^{\circ}$ to the horizontal plane. Draw the plan of the cone, and of its section by a plane parallel to the base and $I \frac{1}{2}$ " from the vertex.

## SECTION B.

*8. Draw a geometrical framework on which the given diaper pattern may be constructed. You are advised to draw the framework geometri-

Q. 8.
cally, and to sketch, freehand, just enough of the pattern to illustrate your meaning. Show very clearly what you consider to be the unit of repeat.
*9. The diagram shows a pattern composed entirely of semicircles. Show two other ways of arranging semicircles so as to produce a repeating pattern, with the necessary constructions for determining centres.
(16)
*10. Show what geometrical help you would use in setting out the plate shown in the diagram. Assume that $A B$, the diameter of the octagon, is $5^{\prime \prime}$.

Q. 9.
*11. Indicate what geometrical constructions you would employ in setting out the circular window shown in the diagram. The "cusping" need not be shown.

*12. Sketch approximate front and side elevations of the chair shown in the diagram.


Pater IV. Q. in.


Paper IV. Q. 12.

## V.

## SECTION A.

*1. The given line $A B$ represents a length of 15 centimetres. Construct a scale by which decimetres, centimetres, and millimetres can be measured up to 3 decimetres. Figure the scale properly, and show by two small marks on it how to take off on it a distance of 263 millimetres.

By means of the scale draw a circle of 125 millimetres' radius, and in it place a chord 189 millimetres long. Write down in millimetres the distance of this chord from the centre of the circle.
( 1 metre $=10$ decimetres, or 100 centimetres, or 1000 millimetres.)
(20)

A B
Q. I.
2. Construct a regular decagon or figure of 10 sides, each side being $1^{\prime \prime}$ long. Within it inscribe five equal circles, each touching two of the others and one side of the decagon.
(N.B.-If a protractor is useỉ for measuring an angle, such use must be clearly shown.)
*3. The curve of the given "scotia" moulding is made up of two quadrants or quarter-circles. Draw the figure from the given dimensions. (N.B. - The diagram is not drawn to scale.)

*4. The diagram shows the leading lines of a window composed of three semicircular-headed " lights" included under a three-centred arch. The side lights are to te each $3^{\prime}$ wide, while the middle light is to be $2^{\prime}$ wide and to have the centre of its semicircle $2^{\prime}$ above those of the other two. Parts of the side semicircles also form part of the enclosing arch. Draw the figure to a scale of $2^{\prime}$ to $\mathrm{I}^{\prime \prime}$. Show clearly how you obtain all points of contact.
(N.B. - The diagram is not drawn to scale.)
5. Two fixed lines $A B$ and $C D$, of indefinite length, cross one another at an angle of $70^{\circ}$. A third line $E F, 3^{\prime \prime}$ long, is movable, so that the end $E$ travels along the line $A B$ while the end $F$ travels along the line $C D$. Draw the complete curve traced by the middle point of $E F$.
(It will be sufficient to find some 12 to 16 points on the curve.)
*6. The diagram gives the fiont elevation of a regular five-pointed star, which is cut out of material $3^{\prime \prime \prime}$ thick. Draw the plan of the star, and also a second elevation on a vertical plane inclined at $60^{\circ}$ to that of the given elevation.

Q. 6.
*7. An elevation is given of a sphere upon which rests a conical lamp-shade. Draw the plan of the shade.


## SECTION B.

*8. Draw with instruments the main geometrical construction lines you would employ in setting out the given pattern. Show plainly what you consider the practical or working unit of repeat.

Q. 8
9. Show how you would arrange a number of circular discs of $\frac{3}{8}^{\prime \prime}$ radius as a diaper pattern -
(i) When each disc touches four others.
(ii) When each disc touches six others.
(iii) When each disc touches three others.

Draw the necessary constructional framework in each case, and show clearly what you consider the unit of repeat in each pattern. ( 20 marks).
*10. Set out carefully the lines of one quarter of the given bookbinding, making your drawing to twice the scale of the diagram.

Q. 10 .
*11. Make a modified version of the border of interlacing circles which the radii shall be $\frac{1_{2}^{\prime \prime}}{2}, \frac{5^{\prime \prime}}{8}, \frac{7^{\prime \prime}}{8}$, and $\mathrm{I}^{\prime \prime}$ respectively. Show at $\mathrm{l}_{1}$ two repeats of the unit.

Q. II.
*12. Sketch approximately, in outline, the plan and the front and side elevations of the workbox and its lid shown in the diagram, omitting all merely ornamental details. Arrange your drawings so as to show how one is projected from another.

Q. 2.

Complete the design of this ruined window, following the indict of every fragment which remains.


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[^0]:    ${ }^{1}$ The line AD is always drawn to the second division from $\mathrm{C}_{3}$ whatever number of sides the polygon may contain.

[^1]:    (For Exercises see p. 232.)

